Name: ______________________________ **Score:** ________/100 **Grade:** _____

The speed round is 30 minutes long. Questions are ordered by difficulty.
Each question is worth 4 points.

1. ________	2. ________	3. ________	4. ________	5. ________
6. ________	7. ________	8. ________	9. ________	10. ________
11. ________	12. ________	13. ________	14. ________	15. ________
16. ________	17. ________	18. ________	19. ________	20. ________
21. ________	22. ________	23. ________	24. ________	25. ________

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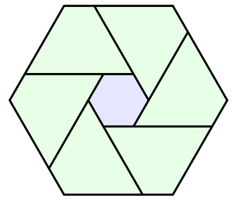
1. What is $2 + 0 + 2 + 6$?

2. In the following figure, the small blue hexagon has an area of 1, and the largest surrounding hexagon has an area of 13. The 6 green trapezoids are all congruent. What is the area of one of the green trapezoids?



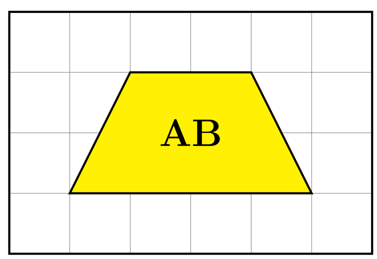
3. If the radius of a circle increases by 20%, by what percentage does its area increase?

4. Justin rolls a pair of fair dice. The probability that the sum of rolls is less than or equal to 4 can be expressed as $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.

5. How many integers from 1 to 15, inclusive, have the same remainder when divided by 2 as when divided by 3?

6. Bob buys gumballs after school to eat as he walks home. The gumball machine he visits contains 5 cherry, 3 lemon, 7 green apple, and 2 grape gumballs. When he inserts a coin, a random gumball drops out. How many coins must he insert into the machine to guarantee that he gets at least 3 differently flavored gumballs?

7. In the diagram below, a point is randomly selected. The probability that it is also within the yellow region "AB" can be written as $\frac{x}{y}$, where x and y are relatively prime positive integers. What is $x + y$?



Note: The point selected can be anywhere inside the 4 by 6 rectangle shown, but it cannot land outside of the rectangle.

8. In ABMC problem writing, one hard problem takes 1 minute to write. It is also known that:

   (a) 3 easy problems take the same amount of time to write as a medium problem
   (b) 2 medium problems take the same amount of time to write as a hard problem
   (c) 7 hard problems take the same amount of time to write as a sadistic problem

   How many seconds would it take to write one easy problem, two medium problems, three hard problems, and four sadistic problems?

9. Ryan and Tommy are dueling in their favorite video game. Tommy has a skill level of 1, while Ryan has a skill level of 100. When they duel, the person with the greater skill level wins. Every time Ryan wins, Ryan will go easier on Tommy and halve his own skill level, while Tommy improves his skill by one level. How many games will they play until Tommy wins his first game (including the game Tommy wins)?

10. There are 10 players on a basketball team, and 5 players have to be chosen for the starting lineup, where the order of the 5 players chosen does not matter. How many ways are there to choose the starting lineup?

11. Eric, Derek, and Fred start running from the same location. Eric runs west at 4 miles per hour, Derek runs north at 5 miles per hour, and Fred runs east at 6 miles per hour. After x hours, Eric is at point E, Derek is at D, and Fred is at F. Find the value of x such that the area of $\triangle DEF$ is 100 square miles.

12. Richard has 35 balls and stacks them into a triangular pyramid. How many balls are in the 2nd layer from the bottom?

13. Define an *interesting* number as any number that is composed of only the digits 6 and 7. Find the units digit of the sum of the squares of all 4 digit interesting numbers.

14. Find the sum of all solutions to the equation $(x^2+3x+1)^{(x^2-17x+60)} = 1$.

15. There are 2026 coins on a table, and each coin is labeled with an integer. The sum of the values of these coins is 22286. A mysterious machine does the following operation:

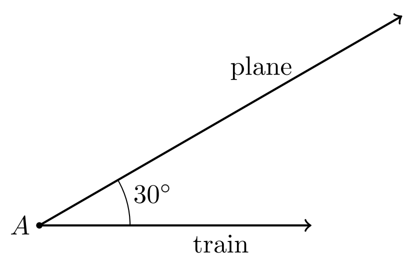
    (a) Pick any two coins. Call the numbers on these coins a and b.

    (b) Replace the coins with new coins valued at $\dfrac{a+69b}{2}$ and $\dfrac{a-67b}{2}$.

    After the machine performs this operation 2048 times, what is the sum of the numbers of the 2026 coins?

16. Find the smallest positive integer $n > 2019$ such that $2^{2020} + 2^{2025} + 2^{2026} + 2^{2027} + 2^n$ is a perfect square.

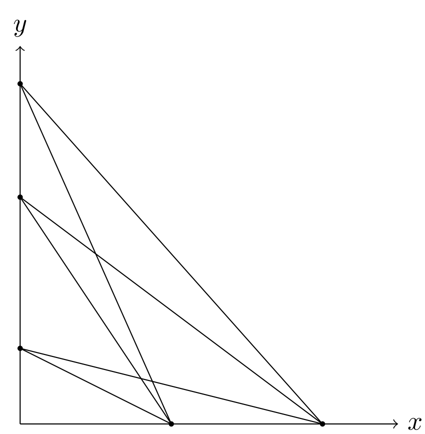
17. At noon, an airplane takes off from point A, maintaining a $30°$ angle with the ground and traveling north. One hour later, a train leaves point A at the same speed and direction as the plane, but parallel to the ground. The time in hours after noon that it will take for the plane to be exactly above the train can be written as $a + b\sqrt{c}$, where a, b, and c are positive integers and $b\sqrt{c}$ is fully simplified. What is $a + b + c$?



18. Let p and q be the roots of the equation $x^2 - 111x + 2026 = 0$. If $\frac{a}{b} = (1 + \frac{1}{p} + \frac{1}{p^2} + \cdots)(1 + \frac{1}{q} + \frac{1}{q^2} + \cdots)$, where a and b are relatively prime positive integers, find $a + b$.

19. There are 10 distinct points on the positive x-axis and 20 distinct points on the positive y-axis, and a line segment is drawn from each point on the positive x-axis to each point on the positive y-axis, making 200 total segments. Given that no three segments are concurrent (except on the axes), find the number of intersection points of the segments with a positive x and y coordinate.

    An example with 3 points on the y -axis and 2 points on the x-axis is shown below:



20. Let $\triangle A_0B_0C_0$ be a triangle with side lengths $A_0B_0 = 1, A_0C_0 = 6$, and $B_0C_0 = \sqrt{37}$. For integers $n \geq 1$, let $\triangle A_nB_nC_n \sim \triangle A_{n-1}B_{n-1}C_{n-1}$ such that $B_nC_n = A_{n-1}C_{n-1}$. What is the sum of the areas of all triangles $A_nB_nC_n$ over all nonnegative integers n?

21. There is exactly one ordered pair of positive integers (a, b) such that $a^2 - 4b^2 = a - 2b + 2^{2026}$, and the ordered pair can be expressed as $(v^w + x, y^z)$ for positive integers v, w, x, y, and z such that v and y are minimized. Compute $v + w + x + y + z$.

22. How many factors of 2026^{2026} have exactly 2026 factors?

23. Compute $\lfloor(\sqrt{11} + 2\sqrt{3})^4\rfloor$.

24. How many ordered pairs (a, b) of positive integers with $4 \leq a \leq b \leq 2026$ exist such that when a apples and b bananas are arranged in a row, the probability that both ends of the row have the same fruit is $\frac{1}{2}$?

25. In $\triangle ABC$, $AB = 15$, $BC = 14$, and $AC = 13$. Suppose the orthocenter of $\triangle ABC$ is H. The lines AH and BH intersect the circumcircle of $\triangle ABC$ at X and Y respectively, such that $X \neq A$ and $Y \neq B$. Given that $XY = \frac{a}{b}$ for relatively prime positive integers a and b, find $a + b$.