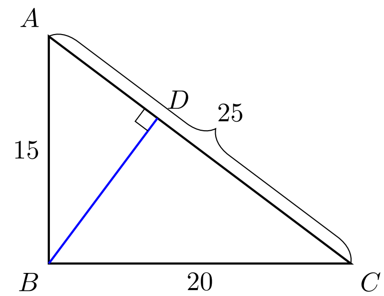
Team Name:________________

Round 1

1. **[9]** We can define the operator ♣ that takes in integers a and b as: $a♣b = a^2 - b^2$. What is the value of the expression $(2♣1) + (4♣3)$?

2. **[9]** In triangle ABC, $AB = 15$, $BC = 20$, and $CA = 25$. D is the foot of the altitude from B to AC. What is the length of BD?



3. **[9]** On Week 1, Richard has 20 coins and Nathan has 10 coins. Each subsequent week, Richard finds 100 coins while Nathan somehow doubles the number of coins he has. If n is the first week in which Nathan's total exceeds Richard's, find the value of n.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 2

1. **[10]** What is the area of a circle that has a circumference of $12\sqrt{\pi}$?

2. **[10]** How many integers less than 2026 are squares of primes?

3. **[10]** Jonathan rolls three fair six-sided dice and adds their results. Let the probability that their sum is even be $\frac{a}{b}$ such that a and b are relatively prime integers. Find $a + b$.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 3

1. **[12]** Benjamin is designing a table for his new club, which he is calling "Knights of the Dodecagonal Table". His table design is made by taking all points with integer coordinates on the graph $x^2+y^2=25$ and connecting them to form a concave polygon. What is the area of this new table, in square units?

2. **[12]** abmNathan and Eric and 43 other people were arguing about how they should slice a cake. Given that 9 people want x slices of cake, 8 people want x^2 slices of cake, 7 people want x^3 slices of cake and so on until 1 person wants x^9 slices of cake, find x if the total number of slices is 2026.

3. **[12]** Given $2^{33}=10^k$, find k, rounded to the nearest integer.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 4

1. **[14]** Suppose $a=20!\cdot 21!\cdot 22!\cdot 23!\cdot 24!\cdot 25!$. If 2^n is the largest power of two that divides a and m^{26} is the largest 26th power that divides a, find $m+n$.

2. **[14]** Find the area of a square inscribed in an equilateral triangle, such that one edge of the square is on the edge of the triangle if the side length of the triangle is $6+4\sqrt{3}$.

3. **[14]** Jonathan and Nathan are playing a card game. Each turn, the player draws one card from a standard 52-card deck plus 2 jokers. If the player draws a King or Queen, they win immediately, but if they draw a Joker, then they lose instantly. Jonathan goes first, then they alternate turns. The probability that Jonathan wins the game can be expressed as $\frac{a}{b}$, where a and b are relatively prime positive integers. What is $a+b$?

1.________________ 2.________________ 3.________________

Team Name:________________

Round 5

1. **[16]** Players 1, 2, 3, and 4 are playing a tennis bracket tournament. They are randomly paired for the first round, and the winners of each game will play each other in the finals. When Player a plays Player b, Player a has a $\frac{a^2+ab}{(a+b)^2}$ chance of winning. However, Player 4 has to play with his left hand for one game, decreasing his skill to the same level that Player 2 has. Player 4 can choose which hand to use based on who he plays first.

   If Player 4 uses the optimal strategy, the probability that Player 4 wins the tournament can be expressed as $\frac{p}{q}$ for relatively prime positive integers p and q. Compute $p+q$.

2. **[16]** An ant is traveling on the coordinate plane. Every second, it moves one unit up, left, right, or down with equal probability. If it starts at $(0,0)$, let $\frac{a}{b}$ be the probability that the ant reaches the point (3,3) at some point in the next 8 seconds. Find $a+b$.

3. **[16]** How many integers from 1 to 2026 divide a number in the form 111...1 (any number of ones). For example, 3 divides 111.

1.________________ 2.________________ 3.________________

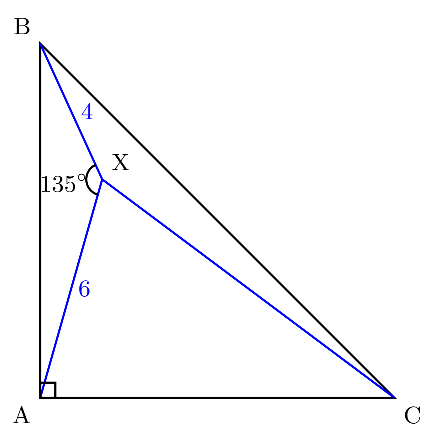
Team Name:________________

Round 6

1. **[18]** There is exactly one real solution to $x^2+2x+1 = \lfloor x^4+x^3+x^2 \rfloor$, and this real solution can be expressed as $a+\sqrt{b}$ for integers a and b. Compute $a+b$.

2. **[18]** Find the sum of all integers n such that 4^n+105 is a perfect square.

3. **[18]** In isosceles right triangle ABC with $AB = AC$, X is located in $\triangle ABC$ such that $AX = 6$, $BX = 4$, and $\angle AXB = 135°$. Compute CX^2.



1.________________ 2.________________ 3.________________

Team Name:________________

Round 7

1. **[21]** Suppose

$$A = \sum_{n=1}^{\infty} \frac{3n^2 + 14n + 10}{n(n+1)(n+2)(n+5)}.$$

Calculate $240A$.

2. **[21]** Alice has 7 cards, numbered 1 through 7. She builds a deck by flipping a fair coin for each card to decide whether to include it. If she includes no cards, the game ends and she scores 0 points; otherwise, she shuffles the chosen cards and deals them out in a row from left to right.

Alice then scores points by counting the number of *decreasing runs* in her row of cards. Reading from left to right, a new run begins with the first card, and a new run begins whenever a card's number is strictly greater than the card immediately before it. For example, if her dealt row is $6, 2, 7, 4, 1$, her cards are grouped into two decreasing runs: $(6, 2)$ and $(7, 4, 1)$, so she scores 2 points.

If the expected number of points Alice scores can be expressed as $\frac{a}{b}$ for relatively prime a, b, compute $a + b$.

Note: A *fair* coin has a $\frac{1}{2}$ chance of landing on either side.

3. **[21]** A hexagon H_1 is inscribed in a square S_1. A square S_2 is then inscribed in H_1. This pattern continues indefinitely, with H_n being inscribed inside of S_n and S_{n+1} being inscribed inside of H_n. Suppose the area of H_k is h_k, and the area of S_k is s_k for every positive integer k. Given that $h_1 s_1 = 24\sqrt{3} - 41$, the maximum possible value of $(h_1 + h_2 + h_3 + \ldots)(s_1 + s_2 + s_3 + \ldots)$ can be expressed as $\frac{a+b\sqrt{c}}{d}$, where a, b, c, and d are positive integers, c is not divisible by the square of any prime, and $\gcd(a, b, d) = 1$. Compute $a + b + c + d$.

1.________________ 2.________________ 3.________________

Team Name:________________

Estimation

[20] Let the *Anirudhian operation*, denoted $\Lambda(n)$, be defined as the product of every prime $p \leq n$. In particular, $\Lambda(n) = \prod_{p \leq n} p$. Using this information, estimate the number of digits of

$$\frac{2026!}{\Lambda(2026)}$$

in base 10. You will earn $\lfloor 20 \min(A/E, E/A)^4 \rfloor$ points, where E is your estimate, and A is the actual answer.

Answer: ________________