Acton-Boxborough Math Competition Online Contest

Saturday, December 13 — Sunday, December 14, 2025

Contest Rules and Format

The 2025 November Contest consists of 15 problems worth 1 point each — each with an answer between 0 and 100,000. The contest window is

Saturday, December 13 to Sunday, December 14.

The following rules supersede any others regarding the Online Contests.

- To submit your responses, fill out a form on our website, abmathcompetitions.org. Include your email, name, school, grade (we require this information for prize purposes).
- Submissions will close at 11:30PM Eastern Standard Time, on the final day of the round.
- Computational aids such as WolframAlpha, calculators, abaci, artificial intelligence, and other related materials are prohibited. Graphing technology such as GeoGebra, Desmos, or a graphing calculator are also prohibited. Drawing aids such as rulers and protractors are permissible. **If you use any of these disallowed materials, you will be disqualified.**
- You may only work individually on the problems—consultation with others is not permitted.
- You may take as much time as you wish during the contest window.
- Ties will be broken by the most difficult problem solved, determined by the number of correct answers to the question. If problem A is solved by a contestants, and problem B is solved by b contestants, with $a < b$, then problem A is said to be "more difficult" than B.

Awards and Prizes

- Top scoring contestants within each round will have their names posted on our website.
- Scores will be emailed to contestants within one week of the contest's end.
- Top scoring contestants across all three rounds will receive prizes.
- Physical prizes (such as AoPS gift certificates, merchandise, calculators, etc.) will be given to top scorers if they attend the ABMC onsite contest.

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Good Luck!

Problems

1. Compute the value of $-\dfrac{2 \cdot 0 + 2 \cdot 5}{2 - 0 + 2 - 5}$.

2. A square has a perimeter of 16. A rectangle with the same perimeter as the square has side lengths in a ratio of $3:1$. Compute the area of the rectangle.

3. A bag has cards labeled 1, 2, 3, 4, and 5. If two cards are randomly selected without replacement, the probability that the sum is even can be expressed as $\frac{a}{b}$ where a and b are relatively prime positive integers. Find $a+b$.

4. What is the 100th digit to the right of the decimal point of $\frac{12}{99}$?

5. Kevin and Eric are bike racing. Eric gives Kevin a 2-mile and 30-minute head start. If Kevin bikes at 15 miles per hour and Eric bikes at 20 miles per hour, how many minutes would Eric take to catch up to Kevin?

6. The product of all factors of 2025 can be written as $a \cdot 2025^b$, where a and b are integers and $a < 2025$. Find $a+b$.

7. Triangle ABC is an isosceles triangle with $AB = AC = 10$, and height AD has length 6. If the median from B intersects AC at the point M, and BM intersects AD at point N. If $\triangle BCN$ is rotated 360° around the line BC, the volume of the resulting two cones formed can be expressed as $\frac{a\pi}{b}$ where a and b are relatively prime positive integers. Find $a+b$.

8. How many positive integer divisors of 20^{25} are perfect cubes but not perfect squares?

9. Let T_n be the n^{th} triangular number defined by $T_n = 1+2+3+\cdots+n$. If $\tau_n = T_1 + T_2 + T_3 + \cdots + T_n$ compute the value of $\tau_1 + \tau_2 + \tau_3 + \cdots + \tau_{25}$.

10. Draw a semi-circle with the equation $f(x) = \sqrt{1-x^2}$. Each second, $f(x)$ will undergo one of the following transformations with a 1/5th probability:

    - Do nothing.
    - Rotate clockwise 90°.
    - Rotate counterclockwise 90°.
    - Double the radius of the semicircle.
    - Halve the radius of the semicircle.

    The probability that $f(x)$ will be at its original position after 5 seconds can be expressed as $\frac{p}{q}$, where p and q are relatively prime. Find $p+q$.

11. Bob has three unfair coins with the following probabilities of landing on heads:

    - Coin 1: $\frac{6}{7}$
    - Coin 2: $\frac{3}{4}$
    - Coin 3: $\frac{1}{3}$

    Bob flips all three coins simultaneously and observes that the total number of heads is at least 2. He then flips all three coins again. Given this observation, the probability that the second flip yields the same total number of heads as the first flip can be expressed as $\frac{a}{b}$ for relativity prime positive integers a and b. Compute $a+b$.

12. Let $f(x)$ be a 5th degree polynomial with the constant term being 5. For $1 \leqslant n \leqslant 5$, let $f(n) = (10-n)f(0)$. Find $(f(6))^2$.

13. There is a right triangular prism $ABCC'A'B'$ with bases $ABC \cong A'B'C'$. $AB = 20$, $AC = 24$, and $BC = 18$. There is a point M on AB such that $AM = 9$ and $BM = 11$. There is a point N on $B'C'$ such that BN is parallel to the plane of $\triangle A'MC$. Given that the length $B'N$ can be expressed as $\frac{a}{b}$ for relativity prime positive integers a and b. Compute $a + b$.

14. Suppose a and b are randomly chosen numbers such that $0 < a < 6$ and $0 < b < 7$. Let $f(n)$ denote the largest integer k such that $3^k \leqslant n$. The probability that $f(a) = f(b)$ can be expressed as $\frac{x}{y}$ where x and y are relatively prime positive integers. Find $x + y$.

15. In isosceles triangle XYZ, $XY = YZ$ and $\angle XYZ = 94°$. P is a point inside $\triangle ABC$ such that $\triangle XYP$ is isosceles and $\angle PXY = 26°$. Find the measure of $\angle XPZ$ in degrees.