Acton-Boxborough Math Competition Online Contest

Saturday, October 18 — Sunday, October 19, 2025

Contest Rules and Format

The 2025 October Contest consists of 15 problems worth 1 point each — each with an answer between 0 and 100,000. The contest window is

Saturday, October 18 to Sunday, October 19.

The following rules supersede any others regarding the Online Contests.

- To submit your responses, fill out a form on our website, abmathcompetitions.org. Include your email, name, school, grade (we require this information for prize purposes).
- Submissions will close at 11:30PM Eastern Standard Time, on the final day of the round.
- Computational aids such as WolframAlpha, calculators, abaci, and other related materials are prohibited. Graphing technology such as GeoGebra, Desmos, or a graphing calculator are also prohibited. Drawing aids such as rulers and protractors are permissible.
- You may only work individually on the problems—consultation with others is not permitted.
- You may take as much time as you wish during the contest window.
- Ties will be broken by the most difficult problem solved, determined by the number of correct answers to the question. If problem A is solved by a contestants, and problem B is solved by b contestants, with $a < b$, then problem A is said to be "more difficult" than B.

Awards and Prizes

- Top scoring contestants within each round will have their names posted on our website.
- Scores will be emailed to contestants within one week of the contest's end.
- Top scoring contestants across all three rounds will receive prizes.
- Physical prizes (such as AoPS gift certificates, merchandise, calculators, etc.) will be given to top scorers if they attend the ABMC onsite contest.

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Good Luck!

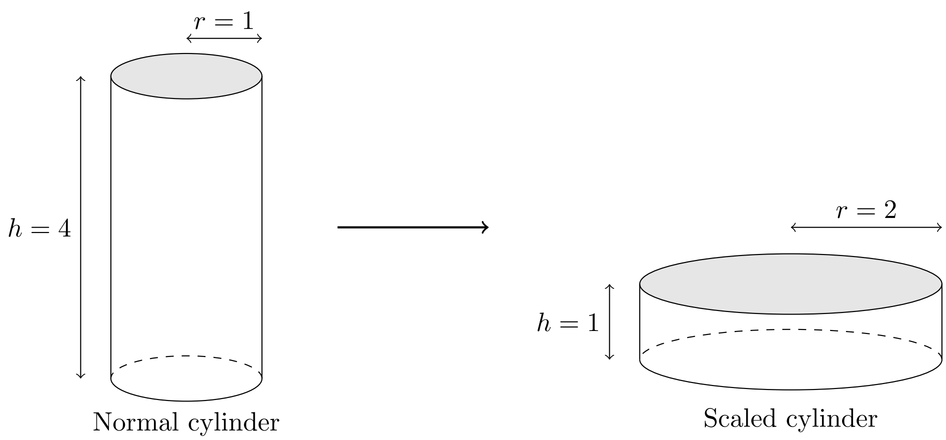
Problems

1. Compute $(2^0 + 2^6) - (2^0 + 2^5)$.

2. Let $S = 2026^2$. Find the product of the non-zero digits of S.

3. Call a cylinder *scalable* if it can be compressed or stretched to be any cylinder with the same volume. Aarush has discovered a scalable cylinder! The current state of Aarush's scalable cylinder has radius 10 and height 30. If the radius of his cylinder when he makes its height double its radius is denoted by r, compute r^3.

   Here is a diagram representing the transformation from $(r, h) = (1, 4)$ to $(r, h) = (2, 1)$:



   Note: The volume of a cylinder can be computed using $\pi r^2 h$ where r is the radius of the base and h is the height of the cylinder.

4. Anirudh is feeding a sugar syrup solution to the hummingbirds outside his house. He begins with 200 mL of a 6% sugar solution. However, he wants 200 mL of a 15% sugar solution. Given that he has access to an infinite amount of 30% sugar solution and will add as much of it as he dumps out of the 6% solution, keeping the volume constant, how many mL of it must he replace?

5. At a party, there are 50 people, and every possible pair of individuals interact exactly once: every person will high-five everyone they know and shake hands with everyone they don't. If a person A knows person B, then person B also knows person A. If among the 50 people:

   - 10 people know nobody else in the party
   - The remaining 40 people each know exactly 5 other people.

   What is the positive difference between the number of high-fives and handshakes?

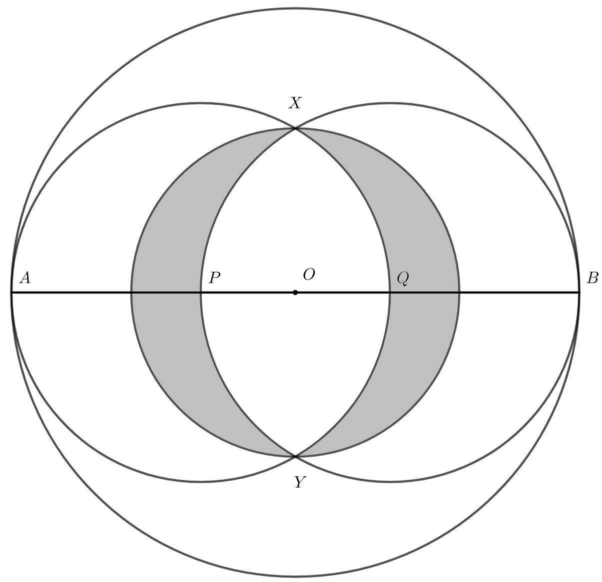
6. If the least common multiple of 6^8, 18^6, 5^{12} and n is 60^{12}, how many distinct values of n are possible?

7. Let A, B, and C be nonzero digits from 1 to 9. Given that $A \div \underline{BB} = 0.\overline{CB}$, find $\underline{ABC}$.

   Note: $\underline{A_1 A_2 \ldots A_n}$ for any n digits represents the n digit number with $A_1, A_2, \cdots,$ and A_n as its digits. For example, if $A_1 = 2$ and $A_2 = 3$, $\underline{A_1 A_2}$ is just the number 23.

8. Let acute $\triangle ABC$ be inscribed in a circle O. Let chord XY of $\odot O$ be parallel and equal to length of BC and intersects AB at M and AC at N such that $XM : MA : NY = 1 : 4 : 2$. Given that the ratio of the area of quadrilateral $XYCB$ to $\triangle AMN$ can be expressed as $\frac{a}{b}$, where a and b are relatively prime integers, find $a + b$.

9. Diameter $\overline{AB}$ of $\odot O_A$ is trisected by points P and Q such that $AP = PQ = QB = 2$. Circles centered at P and Q are drawn internally tangent to $\odot O$. Another circle $\odot O_X$ drawn centered at O and containing points X and Y is drawn. If the shaded area below can be written as $\frac{a}{b}\pi + c\sqrt{d}$, where a, b, c, d are integers, a and b are relatively prime, and $c\sqrt{d}$ is fully simplified, find $a + b + c + d$.



10. Let the sequence $\{a_n\}_{n\in\mathbb{N}}$ be defined by the rule $a_n := n^3 - n + 12$. For each $n \in \mathbb{N}$, we let $d_n = \gcd(a_n, a_{n+1})$. Find the maximum value of d_n.

11. Suppose there are one hundred closed doors, labeled consecutively from 1 to 100. One hundred people pass by the doors in sequence. For each $i \in \{1, 2, \ldots, 100\}$, the i-th person selects a door uniformly at random from the set of doors with labels in $\{1, 2, \ldots, i - 1, i\}$ and opens it. If a door that is already open is selected, it remains open. If the expected number of open doors after the100th person has passed is denoted by ϵ, what is 2ϵ?

12. Consider all the points on a 3-dimensional space with positive integer coordinates. Nathan takes a (not necessarily unique) set of 2026 points with the smallest *Manhattan distances* from the origin. What is the maximum possible Manhattan distance from the origin of one of these points?

    Note: The *Manhattan distance* of two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is equal to $|x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2|$.

13. In triangle ABC, $\angle ABC = 120°$ is bisected by BD. BD is extended to hit the circumcircle of $\triangle ABC$ at point E. Given $AB = 36$ and $BC = 60$, the length of DE can be expressed in the form $\frac{a}{b}$ for some relatively prime integers a, b. Compute $a + b$.

14. In triangle ABC, $\angle BAC = 120°$ and is quadrisected (cut into 4 equal angles). The quadrisectors intersect BC at X, Y, and Z such that B, X, Y, Z, and C lie on a line in that order. Given $AB = 8$ and $AC = 12$, find $a + b$ where $(AB)(AX) + (AX)(AY) + (AY)(AZ) + (AZ)(AC) = a\sqrt{b}$.

15. Ben rolls a fair six-sided die 2025 times. If the probability that the the sum of all results he gets is divisible by 7 is $\frac{m}{6^{2025}}$, then find the last 5 digits of m.