Acton-Boxborough Math Competition Online Contest Solutions

Saturday, November 22 — Sunday, November 23, 2025

1. **Problem:** A rectangle has side lengths 20 and 25. Compute its area.

   Answer: $\boxed{500}$

   Solution: The area of a rectangle is the product of two consecutive sides, and since they are 20 and 25, the area is $\boxed{500}$.

   Proposed by Aarush Kulkarni and Anirudh Pulugurtha, Solution by Shubham Kulkarni

2. **Problem:** Find the smallest positive integer n that is a multiple of 15 and whose digits sum to 15.

   Answer: $\boxed{195}$

   Solution: Because n is a multiple of 15, it has to be a multiple of 5 and 3. Divisibility by 5 means the last digit of n must be either 0 or 5, and Divisibility by 3 means the sum of n's digits must be a multiple of 3.

   The problem states that the digits of n sum to 15, which is divisible by 3. Therefore, n automatically satisfies the divisibility by 3 condition. Thus, we only need to find the smallest integer whose digits sum to 15 and whose last digit is 0 or 5.

   Note that n must be a three digit number, as it is clear that any 2 digit number with unit digit 0 or 5 cannot have its digits sum to 15.

   If the last digit is 5, then the other two digits must sum to 10, thus setting the hundreds digit to 1, and the ten's digit to 9 achieves the minimal value of 195.

   If the last digit is 0, then the other two digits would have to sum to 15, which would be minimized at 690, but $690 > 195$, and so it is not the smallest n.

   Thus the answer is $n = \boxed{195}$.

   Proposed by Anirudh Pulugurtha, Solution by Aarush Kulkarni

3. **Problem:** Jonathan rolls three fair six-sided dice and adds up their results. If the probability that their sum is even is $\frac{a}{b}$, what is $a + b$?

   Answer: $\boxed{3}$

   Solution: To have an even sum, there must be an even number of odd numbers. This means that Jonathan must roll 0 or 2 odd numbers.

   Case 1: All of Jonathan's rolls are odd. This happens with probability $\left(\frac{3}{6}\right)^3 = \frac{1}{8}$.

   Case 2: One of Jonathan's rolls is even and the other two are odd. There are $\binom{3}{1}$ ways to choose which roll is even and the odd rolls are fixed once the even roll is chosen. Thus, the probability that the second case occurs is $\binom{3}{1} \cdot \frac{3}{6} \cdot \left(\frac{3}{6}\right)^2 = \frac{3}{8}$.

   Summing the probabilities for these two cases, we get $\frac{1}{8} + \frac{3}{8} = \frac{1}{2}$, making our final answer $1 + 2 = \boxed{3}$.

   Proposed by Jonathan Ren, Solution by Christina Lu

4. **Problem:** A year is deemed *uncommon* if it is both a perfect square and a multiple of five. If 2025 is an *uncommon* year, then in how many more years will the next *uncommon* year be?

   Answer: $\boxed{475}$

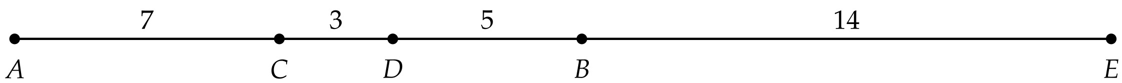
   Solution: Since 2025 is 45^2, the next possible year where it will be both a multiple of 5 and a perfect square will be $50^2 = 2500$, thus this answer is $2500 - 2025 = \boxed{475}$.

   Proposed by Shubham Kulkarni, Solution by Shubham Kulkarni

5. **Problem:** Points A, B, C, D, and E lie on a straight line, not necessarily in order. If the distance from A to C is 7, the distance from B to D is 5, the distance from B to E is 14, and the distance from B to C is 8, what is the longest possible distance between 2 points?

   Answer: $\boxed{29}$

   Solution:



   The ideal solution to this problem would take the 4 known lengths and lay them out for the longest possible line segment, but since two of the lengths have endpoint B, they must extend in opposite directions from B, meaning only one can contribute to the overall length of the segment.

   We start by laying out segment $\overline{AC}$ and segment $\overline{CB}$ such that they are connected via point C. Our new line segment, segment $\overline{AB}$, has length $7 + 8 = 15$. Since segments $\overline{BD}$ and $\overline{BE}$ must be on different sides of B, we choose the longer segment $\overline{BE}$ to be placed further from the endpoint A, while $\overline{BD}$ is placed such that point D is between points C and B. The final order of the points along the line is A, C, D, B, E. Thus, our overall segment length and solution to the problem is $7 + 8 + 14 = \boxed{29}$.

   Proposed by Ryan Xia, Solution by Ryan Xia

6. **Problem:** Ten problem writers in ABMC sit at a circular table during a meeting. If Tanish, Eric, and Nathan all sit in consecutive spots in the circle, how many ways are there for the ten people to arrange themselves (there are no clones in ABMC)?

   Answer: $\boxed{30240}$

   Solution: Note that we can treat Tanish, Eric, and Nathan as an individual "block", find the number of ways we can arrange them three, and then find the number of ways we can arrange the remaining problem writers.

   In those three seats, there is $3 \cdot 2 \cdot 1 = 6$ to shift the seats of the people (3 for the first person, 2 for the second, 1 for the final).

   Finally, the number of ways to arrange the remaining seven people is $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$.

   Combining both, we get $6 \cdot 5040 = \boxed{30240}$.

   Proposed by Nathan Tan, Solution by Shubham Kulkarni

7. **Problem:** Thomas was given two terms of the infinite arithmetic sequence s with first term 27:

$$s = \{27, a_2, \cdots\}.$$

   Thomas knows the value of a_2, which is not 27, and his task is to find the third term of the sequence. However, he accidentally calculates the next term assuming s was geometric.

   Thomas's incorrect answer is equal to the sixth term of s. Find a_2, the second term of s.

   Answer: $\boxed{108}$

   Solution: If Thomas assumes that s is geometric, then the sequence has a common ratio of $\frac{a_2}{27}$; therefore, the term he calculates is $\frac{a_2^2}{27}$. The correct arithmetic sequence has a common difference of $a_2 - 27$; thus, the sixth term would be $27 + 5(a_2 - 27) = 5a_2 - 108$. This implies that

$$\frac{a_2^2}{27} = 5a_2 - 108,$$

which is equivalent to $a_2^2 - 135a_2 + 108 \cdot 27 = 0$ or

$$(a_2 - 108)(a_2 - 27) = 0.$$

By the condition given that $a_2 \neq 27$, we get the desired answer $a_2 = \boxed{108}$.

Proposed by Daniel Ren, Solution by Steven Feng

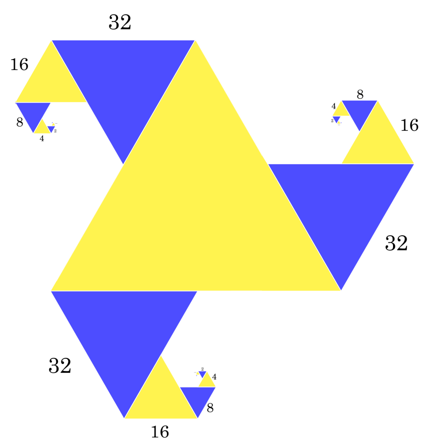
8. **Problem:** An artist designs a fractal pattern called the "Dragon's Scale" using a sequence of equilateral triangles. The construction process is as follows:

   (a) Begin with a central equilateral triangle, T_0, whose side length is 64 units.

   (b) From each of the three vertices of T_0, a spiral of n smaller equilateral triangles is constructed outwards. Let's call these the "arms" of the fractal.

   (c) The triangles in each arm, denoted $T_1, T_2, \ldots, T_n$, have side lengths that follow a geometric progression. The side length of triangle T_k, for $k \in \{1, \ldots, n\}$, is half the side length of triangle T_{k-1}.

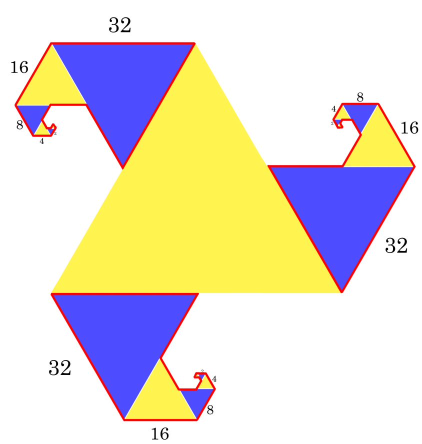
   The resulting diagram, showing the central triangle T_0 and the $n =$ triangles of each of the three arms, is shown below.



   What is the perimeter of the completed fractal pattern?

   Answer: $\boxed{384}$

   Solution: It suffices to split the diagram into three equivalent parts as shown below, compute the perimeter of each equivalent part, sum over the three computed values, then add back the contribution from the central triangle to find our desired answer. Observe the below figure, which splits the diagram from problem into three equivalent parts,



The central triangle obviously contributes $3(32) = 96$ to the sum. Let s_k be the side length of triangle T_k for $k \geqslant 0$. We are given $s_0 = 64$, and for $k \geqslant 1$, the side length s_k is half of s_{k-1}. This implies that the side lengths of the triangles in each arm follow the geometric progression $s_k = 64 \cdot 2^{-k}$.

The total perimeter of the fractal is the sum of the perimeters of the three outwardly spiraling arms. Let P_{arm} denote the perimeter contributed by a single arm. The total perimeter is then $96 + 3 \cdot P_{\text{arm}}$.

The perimeter of a single arm is $P_{\text{arm}} = 96$. For each triangle T_k (for $k \geqslant 1$) in an arm, two of its sides are exposed, contributing to the total perimeter. The contribution from triangle T_k is therefore $2s_k$. The total perimeter of one arm is the sum of these contributions over all triangles in the infinite spiral (note that for the case with side length 32, there is only one of these, which we remove from the sum),

$$P_{\text{arm}} = 32 + \sum_{k=1} 2s_k = 32 + \sum_{k=1} 2(32 \cdot 2^{-k}) = 32 + 64 \sum_{k=1} \left(\frac{1}{2}\right)^k.$$

The infinite sum is a geometric series with first term $a = 1/2$ and common ratio $r = 1/2$. The value of this sum is

$$\sum_{k=1} \left(\frac{1}{2}\right)^k = \frac{a}{1-r} = \frac{1/2}{1-1/2} = 1.$$

This implies that the perimeter of one arm is

$$P_{\text{arm}} = 32 + 64 \cdot 1 = 96.$$

Thus,

$$P_{\text{total}} = 96 + 3 \cdot P_{\text{arm}} = 96 + 3 \cdot 96 = \boxed{384}.$$

Proposed by Aarush Kulkarni, Solution by Aarush Kulkarni and Jonathan Ren

9. **Problem:** Anirudh works at "Acton-Boxborough Fries," a restaurant known for its 41 distinct fry sizes, labeled $\{1, \ldots, 41\}$. The price of each size is a positive odd integer, and a larger size always costs more than a smaller one.

When a customer pays for an order, Anirudh has an interesting way of bagging the fries. For a single payment, he follows a strict rule:

(a) He finds the largest size of fries the customer can afford with their remaining money and puts it in the bag.

(b) He repeats this process, but he will now only consider sizes that are strictly smaller than the one he just bagged.

Anirudh wants to use this system to create a massive personal collection of fries by placing multiple orders. His target collection must have zero bags of size 41. For every other size k (from 1 to 40), the number of size-k bags he owns must be exactly one more than the total number of bags he has of all sizes larger than k.

If the total number of fry bags in Anirudh's completed collection can be written as $a^b - c$, where a, b, c are positive integers, what is the value of $a + b + c$?

Answer: $\boxed{43}$

Solution: The key to this problem is working out the first few cases using an engineers induction approach.

Let n_k be the number of bags of size k in Anirudh's collection for $k \in \{1, \ldots, 41\}$. According to the problem statement, Anirudh has 0 bags of size 41, so $n_{41} = 0$. For any other size $1 \leqslant k \leqslant 40$, the number of bags of size k is 1 more than the total number of bags less than size k. In particular,

$$n_k = 1 + \sum_{j=k+1}^{41} n_j.$$

Now, let us test out some values,

- For size 40, the rule gives $n_{40} = 1 + n_{41} = 1$.
- For size 39, the rule gives $1 + n_{40} + n_{41} = 1 + 1 + 0 = 2$.
- For size 38, the rule gives $1 + n_{39} + n_{40} + n_{41} = 2 + 1 + 1 + 0 = 4$.
- And in general, for size $k < 40$, we have,

$$n_k = 2^{40-k}.$$

Now we can find the total number of bags by summing them all up.

$$\begin{aligned} T = \sum_{k=1}^{41} n_k = n_{41} + \sum_{k=1}^{40} n_k = 0 + \sum_{k=1}^{40} 2^{40-k} &= (2^{40-1} + 2^{40-2} + \cdots + 2^{40-40}) \\ &= 2^{39} + 2^{38} + \cdots + 2^{0}. \\ &= \frac{1(2^{40}-1)}{2-1} = 2^{40} - 1, \end{aligned}$$

and thus our desired answer is $2 + 40 + 1 = \boxed{43}$.

Remark. We can prove the pattern we found. For any size $k < 40$, we have

$$n_k = 1 + n_{k+1} + n_{k+2} + \cdots + n_{41} = 1 + n_{k+1} + \sum_{j=k+2}^{41} n_j.$$

The rule for size $k+1$ is $n_{k+1} = 1 + \sum_{j=k+2}^{41} n_j$. We can rearrange this to get $\sum_{j=k+2}^{41} n_j = n_{k+1} - 1$. Substituting this into our expression for n_k, we find

$$n_k = 1 + n_{k+1} + (n_{k+1} - 1) = 2n_{k+1}.$$

Thus, for any size k from 1 to 39, the number of bags of size k is exactly double the number of bags of size $k+1$.

Proposed by Aarush Kulkarni, Solution by Aarush Kulkarni

10. **Problem:** Aarush rolls 5 standard 6-sided dice. For each of the 5 rolls, he gets a stick of length the number he rolled (Ex. if Aarush rolls a 4, he gets a stick of length 4). Let the probability that Aarush can take 4 of the 5 sticks and make a rectangle be $\frac{a}{b}$, where a and b are relatively prime numbers. What is $a + b$?

Answer: $\boxed{209}$

Solution: For 4 out of the 5 sticks to make a rectangle, there must be 2 disjoint pairs of dice that roll the same number. For example, the rolls could be $(4, 4, 4, 4, 5)$ or $(1, 1, 1, 1, 1)$ or $(2, 2, 3, 6, 6)$ but not $(1, 2, 2, 3, 4)$. There are 4 cases where this occurs:

Case 1: All 5 dice roll the same number. In this case, there are 6 ways to choose the number that all 5 of the dice roll and only 1 distinct way to order the dice. Multiplying, we get $6 \cdot 1 = 6$.

Case 2: 4 dice roll the same number, and 1 die rolls a second number. In this case, there are $\binom{5}{4}$ ways to order the dice, and $6 \cdot 5$ ways to choose the colors. Multiplying, we get $\binom{5}{4} \cdot 6 \cdot 5 = 150$.

Case 3: 2 dice roll one number and 3 dice roll a second number. In this case, there are $\binom{5}{3}$ ways to order the dice, and $6 \cdot 5$ ways to choose the colors. Multiplying, we get $\binom{5}{3} \cdot 6 \cdot 5 = 300$.

Case 4: 2 dice roll one number, 2 dice roll a second number, and 1 die rolled a third number. In this case, there are $\frac{5!}{2! \cdot 2!}$ ways to order the dice, and $6 \cdot 5 \cdot 4$ ways to choose the colors. However, multiplying these numbers overcounts every combination of rolls once [1]. Thus, we need to divide by 2, which gives us $\frac{\left(\frac{5!}{2!\cdot 2!}\right)\cdot(6\cdot 5\cdot 4)}{2} = 1800$.

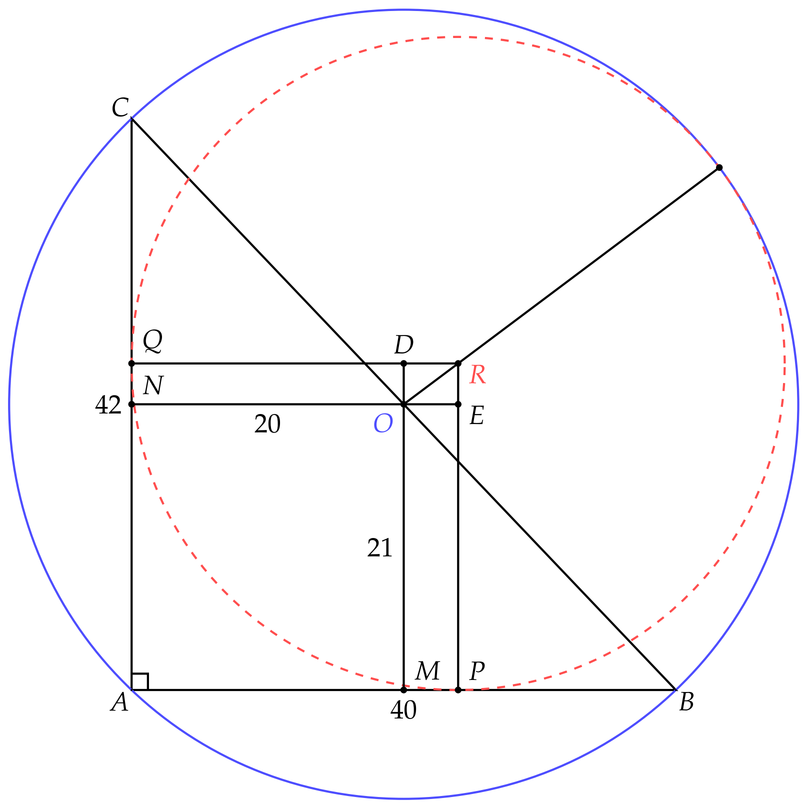
The total probability is $\frac{6+150+300+1800}{6^5} = \frac{2256}{7776} = \frac{47}{162}$, making our final answer $47 + 162 = \boxed{209}$.

Proposed by Tanish Parida, Solution by Christina Lu

11. **Problem:** Right triangle $\triangle ABC$ with legs $AB = 40$ and $AC = 42$ is drawn with circumcircle ω. The unique circle ξ tangent to $\overline{AB}$ and $\overline{AC}$, as well as internally tangent to ω, is drawn with radius r. Find r^2.

Answer: $\boxed{576}$

Solution:



Let O and R be the centers of circles ω and ξ, respectively.

Introduce points D and E so that $ODRE$ forms a rectangle, with D on the perpendicular from O to $\overline{AB}$ and E on the perpendicular from O to $\overline{AC}$. We will label the intersections of $\overline{OD}$ and $\overline{RE}$ with $\overline{AB}$ as M and P, respectively, and we will likewise label the intersections of $\overline{RD}$ and $\overline{EO}$ with $\overline{AC}$ as N and Q, respectively. Notice that M and N are the midpoints of $\overline{AB}$ and $\overline{AC}$, respectively.

Since $RQ = r$ and $DQ = MA = \frac{40}{2} = 20$, $RD = RQ - DQ = r - 20$ (r is the radius of ξ).

Similarly, since $RP = r$ and $EP = NA = \frac{42}{2} = 21$, $RE = RP - EP = r - 21$.

The hypotenuse $\triangle ABC$ has length $\sqrt{40^2 + 42^2} = 58$, so the circumradius of ω is $\frac{58}{2} = 29$. Since ξ is internally tangent to ω, the distance $\overline{OR}$ equals $29 - r$.

[1]To see this, let $x, y \in \{1, 2, 3, 4, 5, 6\}$ such that $x \neq y$. Order the 5 dice. Let one pair of equal dice roll x and the other pair of equal dice roll y, and label the pairs "first" and "second," respectively (based on the actual dice, not the value they rolled). We can reorder the dice by swapping the positions of the first and second pairs. Then, if the second pair rolls x and the first pair rolls y, the new arrangement of dice is equivalent to the original arrangement. Hence, we have overcounted every arrangement of dice once.

Applying the Pythagorean Theorem in rectangle $ODRE$ gives:

$$\begin{aligned}(r-20)^2+(r-21)^2&=(29-r)^2\\ r^2-40r+20^2+r^2-42r+21^2&=29^2-58r+r^2\\ r^2-24r&=0\\ r(r-24)&=0.\end{aligned}$$

r cannot be 0, thus $r=24$, and $r^2=\boxed{576}$.

Proposed by Daniel Ren, Solution by Ryan Xia and Eric Shi Chen

12. **Problem:** Let $f(x)$ be a function such that $\frac{f(x)}{f(\frac{2025}{x})}=k_2x^2+k_1x$ for all $x>0$. Given that the sum of all possible values of k_1+k_2 can be expressed in the form $\frac{a}{b}$, where a and b are relatively prime positive integers, find $a+b$.

Answer: $\boxed{2071}$

Solution: Let $P(x)=k_2x^2+k_1x$. We work with the generalized functional equation for any C, $\frac{f(x)}{f(C/x)}=P(x)$, where in this problem, $C=2025$. Substituting C/x for x implies $\frac{f(C/x)}{f(x)}=P(C/x)$. Multiplying these relations, we obtain $P(x)P(C/x)=1$.

The next key part to the problem is discovering that $k_1k_2=0$ and $k_2^2C^2+k_1^2C-1=0$. Note that by the condition on k_1+k_2, we have

$$\begin{aligned}(k_2x^2+k_1x)\left(k_2\frac{C^2}{x^2}+k_1\frac{C}{x}\right)&=1\\ (k_2x^2+k_1x)(k_2C^2+k_1Cx)&=x^2\\ k_1k_2Cx^3+(k_2^2C^2+k_1^2C-1)x^2+k_1k_2C^2x&=0\end{aligned}$$

For this identity to hold for all $x>0$, all coefficients must be zero. Since $C\neq 0$, the x^3 and x terms imply $k_1k_2=0$. The x^2 term implies $k_2^2C^2+k_1^2C=1$. Since $C=45^2$, we consider two cases.

Case 1: $k_2=0$. Then $k_1^2C=1$, so $k_1=\pm1/\sqrt{C}=\pm1/45$. The possible values for k_1+k_2 are $\frac{1}{45}$ and $-\frac{1}{45}$.

Case 2: $k_1=0$. Then $k_2^2C^2=1$, so $k_2=\pm1/C=\pm1/2025$. The possible values for k_1+k_2 are $\frac{1}{2025}$ and $-\frac{1}{2025}$.

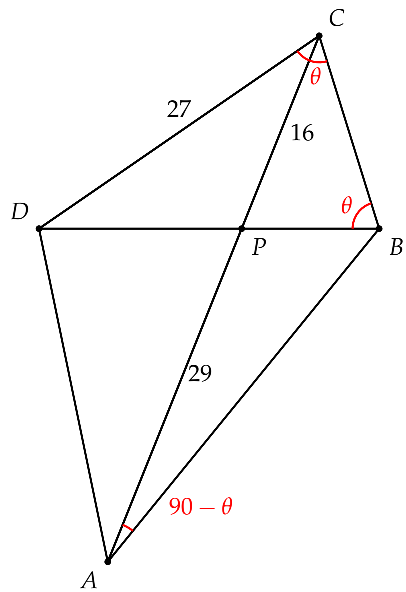
The sum of all possible positive values for k_1+k_2 is

$$S=\frac{1}{45}+\frac{1}{2025}=\frac{46}{2025}.$$

Thus, the answer is $a+b=46+2025=\boxed{2071}$.

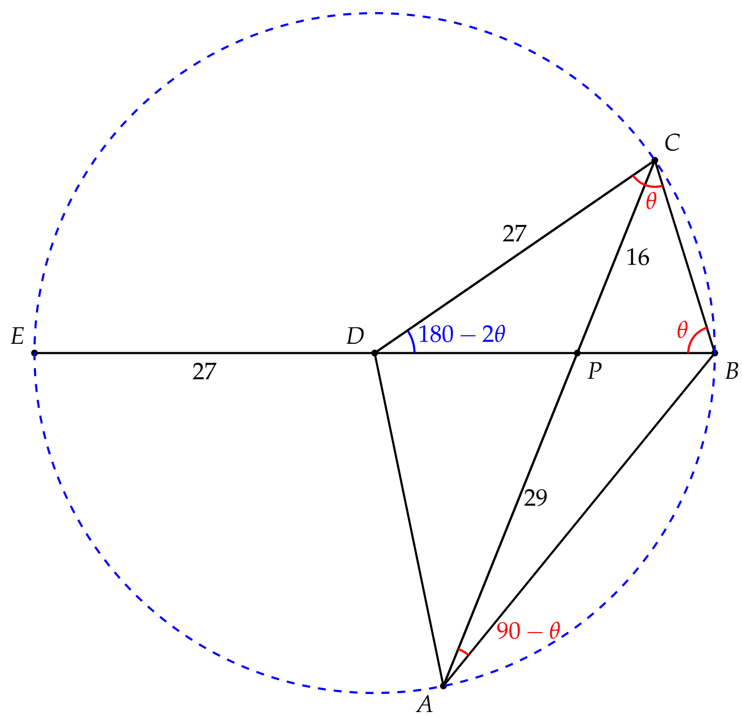
Proposed by Eric Li, Solution by Aarush Kulkarni

13. **Problem:** Given the following diagram, compute DP^2.



Answer: 265

Solution:



We begin by proving that the circumcenter of $\triangle ABC$ is the point D.

Observe that $\angle CDB = 180^\circ - 2\theta$, which is exactly twice the measure of $\angle CAB$.

Since $\triangle DCB$ is isosceles, consider a circle centered at D passing through C and B. This circle must also pass through A, because the angle subtended by an arc at the center of a circle is twice the angle subtended by it at any point on the remaining part of the circle.

In this circle, $\angle CDB$ is the angle subtended by $\widehat{BC}$ at the center of a circle, and $\angle CAB$ is the angle subtended by $\widehat{BC}$ at point A on the circle.

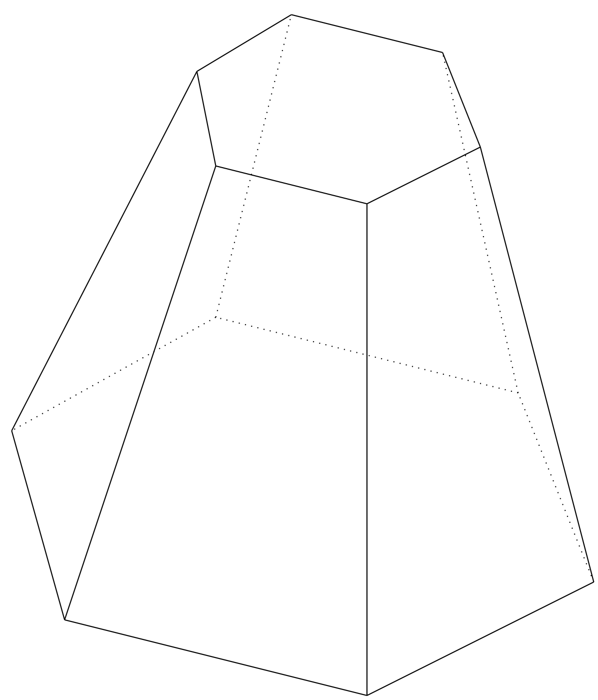
Now, we can use power of a point to find DP. Extending BD to E such that BE is the diameter, we can now apply power of a point on point P:

$$\begin{aligned} EP \cdot PB &= CP \cdot PA \\ (27 + DP)(27 - DP) &= 16 \cdot 29 \\ 729 - DP^2 &= 464 \\ DP^2 &= \boxed{265}. \end{aligned}$$

Proposed by Eric Shi Chen, Solution by Eric Shi Chen

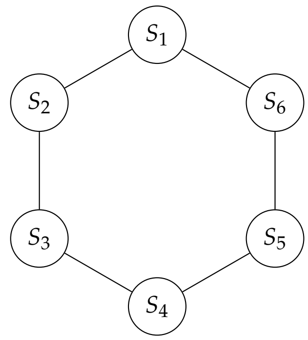
14. **Problem:** The base of Aarush's American Revolution Monument Project is a hexagonal truncated pyramid. Aarush has 5 colors of paint and wants to paint each of the 8 faces such that no two adjacent faces are the same color. In how many ways can he paint the base of his monument?

    (Two faces are considered adjacent if they share an edge. Colorings that differ only by rotations are considered distinct. Refer to the diagram below, but note that it is not drawn to scale.)



Answer: $\boxed{4980}$

Solution: Let the 3D shape have a top face denoted T and a bottom face denoted B, and six side faces $S_1, S_2, \ldots, S_6$ arranged in a cycle configuration. The adjacencies between faces can be represented by a graph where each face is a vertex and an edge connects two vertices if the corresponding faces are adjacent. The below figure is the cycle graph; note that is does not include the top and bottom faces.



Our strategy is to first color the top and bottom faces, and do casework on whether they are the same or different colors. The number of available colors for the side faces, which form a cycle, depends on this choice.

If T and B have the same color, this leaves 4 colors available for the 6-cycle (a cycle of 6 vertices) of side faces. If T and B have different colors, this leaves 3 colors available for the 6-cycle.

Therefore, to solve the problem, we must first calculate the number of ways to color a 6-cycle with 4 and 3 colors, respectively. Let $N_n(k)$ denote the number of ways to properly color a cycle of n vertices with k colors. Our goal is to compute $N_6(4)$ and $N_6(3)$. In order to compute these values, we can establish a recurrence for $N_n(k)$ through the following three steps.

(a) A proper coloring (a coloring where no two adjacent vertices have the same color) of an n-cycle is equivalent to a proper coloring of an n-vertex path where the endpoints have different colors. Let $D_n(k)$ be the number of ways to color an n-vertex path with different endpoint colors, and $E_n(k)$ be the number of ways with the same endpoint colors. It is clear that $N_n(k) = D_n(k)$.

(b) Now, we consider coloring a path of $n+1$ vertices. To ensure the endpoints have different colors, we can start with a valid coloring of the first n vertices. If the endpoints of the n-path have the same color (in $E_n(k)$ ways), the $(n+1)$-th vertex can have any of the other $k-1$ colors.

   If the endpoints of the n-path have different colors (in $D_n(k)$ ways), the $(n+1)$-th vertex must avoid two colors, leaving $k-2$ choices. This implies the recurrence $D_{n+1}(k) = (k-1)E_n(k) + (k-2)D_n(k)$.

   Furthermore, a coloring of an n-path with the same endpoint colors is constructed by taking a coloring of an $(n-1)$-path with different endpoint colors and setting the color of the n-th vertex to be that of the first. This means $E_n(k) = D_{n-1}(k)$. Substituting this into the recurrence, we obtain $D_{n+1}(k) = (k-2)D_n(k) + (k-1)D_{n-1}(k)$. Therefore, for $n \geqslant 3$, the number of colorings for a cycle follows the recurrence

$$N_{n+1}(k) = (k-2)N_n(k) + (k-1)N_{n-1}(k).$$

   For the base cases, a 2-cycle has $N_2(k) = k(k-1)$ colorings. A 3-cycle (a triangle) has $N_3(k) = k(k-1)(k-2)$ colorings.

(c) Now we compute the required values for $N_6(k)$. For $k = 4$, we recursively compute to get 732. For $k = 3$, we follow a similar recipe to get 66.

Now, we finish by simple casework on the top and bottom faces.

Case 1: T and B have the same color. There are 5 choices for this common color. The side faces cannot use this color, so they must be colored using the remaining 4 colors. The number of ways to color the side faces is $N_6(4) = 732$. Thus, the total ways for this case is $5 \times 732 = 3660$.

Case 2: T and B have different colors. There are $5 \times 4 = 20$ ways to choose the colors for T and B. The side faces cannot use these two colors, so they must be colored using the remaining 3 colors. The number of ways to color the side faces is $N_6(3) = 66$. Thus, the total ways for this case is $20 \times 66 = 1320$.

Thus, the answer is $3660 + 1320 = \boxed{4980}$.

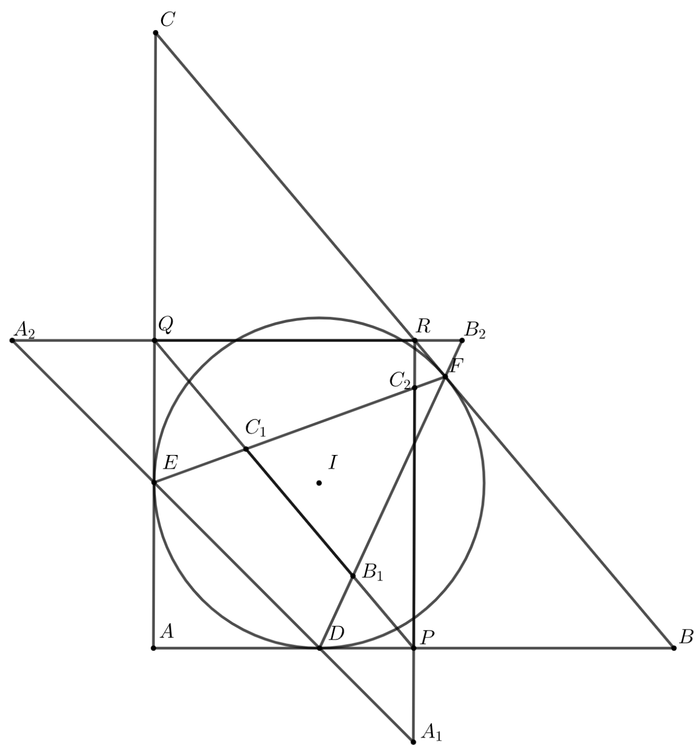
Remark. In fact, we can generalize this to any k colors for $k \geqslant 3$. The number of ways to properly color a graph G with k colors, the *chromatic number*, is given by a function $\chi(G, k)$, which is always a polynomial in k called the *chromatic polynomial*, which is a polynomial where if you plug in a value k, it outputs the chromatic number. The value of this polynomial at an integer k gives the number of proper k-colorings (a coloring of a graph where no two colors are adjacent) of G. In this case, the number of ways to color a cycle graph C_n with k colors is given by the formula

$$\chi(C_n, k) = (k-1)^n + (-1)^n(k-1).$$

For our problem, the number of ways to color the side faces with 4 available colors is $\chi(C_6, 4) = (4-1)^6 + (-1)^6(4-1) = 3^6 + 3 = 729 + 3 = 732$. The number of ways to color the side faces with 3 available colors is $\chi(C_6, 3) = (3-1)^6 + (-1)^6(3-1) = 2^6 + 2 = 64 + 2 = 66$. The total number of colorings is therefore $5 \cdot \chi(C_6, 4) + 5 \cdot 4 \cdot \chi(C_6, 3) = 5(732) + 20(66) = 3660 + 1320 = 4980$.

Proposed by Christina Lu, Solution by Aarush Kulkarni

15. **Problem:** Triangle ABC has incircle ω and incenter I, and side lengths $AB = 44$, $AC = 52$, and $BC = 68$. Let the feet of the altitudes from I to AB, AC, and BC be D, E, and F, and the midpoints of AB, AC, and BC be P, Q, and R. Let the intersections of DE with PR and QR be A_1 and A_2, the intersections of DF with PQ and QR be B_1 and B_2, and the intersections of EF with PQ and PR be C_1 and C_2. Find $A_1R + A_2R + B_1Q + B_2Q + C_1P + C_2P$.



Answer: $\boxed{164}$

Solution: Since A_1 lies on line PR, we have that $A_1P \parallel AE$, and thus it follows that $\angle PDA_1 = \angle ADE$ and $\angle DA_1P = \angle DEP$ so we have that $\triangle AED \sim \triangle PA_1D$.

Since AD and AE are both tangent to the incircle, we have that $AE = AD$ and therefore $PD = PA_1$.

Let $AD = AE = x$, $BD = BR = y$, and $CE = CR = z$. We have that $x + y + z = \frac{44+52+68}{2} = 82$. We also have $x + y = 44$, $x + z = 52$, and $y + z = 68$. Solving gives us that $x = 14$, $y = 30$, and $z = 38$.

Thus, $RP = \frac{AC}{2} = 26$ and $A_1P = DP = AP - AD = 22 - 14 = 8$. We also have $A_2Q = EQ = AQ - EQ = 26 - 14 = 12$ and $QR = \frac{AB}{2} = 22$, so we have that $A_2R + A_1R = A_1P + AR + A_2Q + QR = 68$. Similarly, we can use this strategy to find $B_1Q + B_2Q = 52$ and $C_1P + C_2P = 44$, so we have that

$$A_1R + A_2R + B_1Q + B_2Q + C_1P + C_2P = 52 + 44 + 68 = \boxed{164}.$$

Proposed by Raymond Gao, Solution by Benjamin Li