Acton-Boxborough Math Competition Online Contest

Saturday, November 22 — Sunday, November 23, 2025

Contest Rules and Format

The 2025 November Contest consists of 15 problems worth 1 point each — each with an answer between 0 and 100,000. The contest window is

Saturday, November 22 to Sunday, November 23.

The following rules supersede any others regarding the Online Contests.

- To submit your responses, fill out a form on our website, abmathcompetitions.org. Include your email, name, school, grade (we require this information for prize purposes).
- Submissions will close at 11:30PM Eastern Standard Time, on the final day of the round.
- Computational aids such as WolframAlpha, calculators, abaci, and other related materials are prohibited. Graphing technology such as GeoGebra, Desmos, or a graphing calculator are also prohibited. Drawing aids such as rulers and protractors are permissible. **If you use any of these disallowed materials, you will be disqualified.**
- You may only work individually on the problems—consultation with others, **including but not limited to artificial intelligence,** is not permitted.
- You may take as much time as you wish during the contest window.
- Ties will be broken by the most difficult problem solved, determined by the number of correct answers to the question. If problem A is solved by a contestants, and problem B is solved by b contestants, with $a < b$, then problem A is said to be "more difficult" than B.

Awards and Prizes

- Top scoring contestants within each round will have their names posted on our website.
- Scores will be emailed to contestants within one week of the contest's end.
- Top scoring contestants across all three rounds will receive prizes.
- Physical prizes (such as AoPS gift certificates, merchandise, calculators, etc.) will be given to top scorers if they attend the ABMC onsite contest.

Thanks to our Sponsors!

Good Luck!

Problems

1. A rectangle has side lengths 20 and 25. Compute its area.

2. Find the smallest positive integer n that is a multiple of 15 and whose digits sum to 15.

3. Jonathan rolls three fair six-sided dice and adds up their results. If the probability that their sum is even is $\frac{a}{b}$, what is $a + b$?

4. A year is deemed *uncommon* if it is both a perfect square and a multiple of five. If 2025 is an *uncommon* year, then in how many more years will the next *uncommon* year be?

5. Points A, B, C, D, and E lie on a straight line, not necessarily in order. If the distance from A to C is 7, the distance from B to D is 5, the distance from B to E is 14, and the distance from B to C is 8, what is the longest possible distance between 2 points from the five?

6. Ten problem writers in ABMC sit at a circular table during a meeting. If Tanish, Eric, and Nathan all sit in consecutive spots in the circle, how many ways are there for the ten people to arrange themselves (there are no clones in ABMC)?

7. Thomas was given two terms of the infinite arithmetic sequence s with first term 27:

$$s = \{27, a_2, \cdots\}.$$

Thomas knows the value of a_2, which is not 27, and his task is to find the third term of the sequence. However, he accidentally calculates the next term assuming s was geometric.

Thomas's incorrect answer is equal to the sixth term of s. Find a_2, the second term of s.

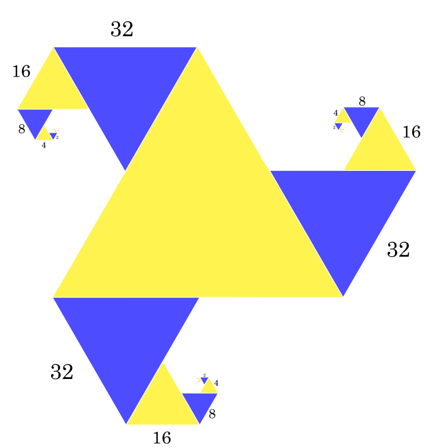
8. An artist designs a fractal pattern called the "Dragon's Scale" using a sequence of equilateral triangles. The construction process is as follows:

   (a) Begin with a central equilateral triangle, T_0, whose side length is 64 units.

   (b) From each of the three vertices of T_0, a spiral of n smaller equilateral triangles is constructed outwards. Let's call these the "arms" of the fractal.

   (c) The triangles in each arm, denoted $T_1, T_2, \ldots, T_n$, have side lengths that follow a geometric progression. The side length of triangle T_k, for $k \in \{1, \ldots, n\}$, is half the side length of triangle T_{k-1}.

The resulting diagram, showing the central triangle T_0 and the $n = \infty$ triangles of each of the three arms, is shown below.



What is the perimeter of the completed fractal pattern?

9. Anirudh works at "Acton-Boxborough Fries," a restaurant known for its 41 distinct fry sizes, labeled $\{1, \ldots, 41\}$. The price of each size is a positive odd integer, and a larger size always costs more than a smaller one.

   When a customer pays for an order, Anirudh has an interesting way of bagging the fries. For a single payment, he follows a strict rule:

   (a) He finds the largest size of fries the customer can afford with their remaining money and puts it in the bag.

   (b) He repeats this process, but he will now only consider sizes that are strictly smaller than the one he just bagged.

   Anirudh wants to use this system to create a massive personal collection of fries by placing multiple orders. His target collection must have zero bags of size 41. For every other size k (from 1 to 40), the number of size-k bags he owns must be exactly one more than the total number of bags he has of all sizes larger than k.

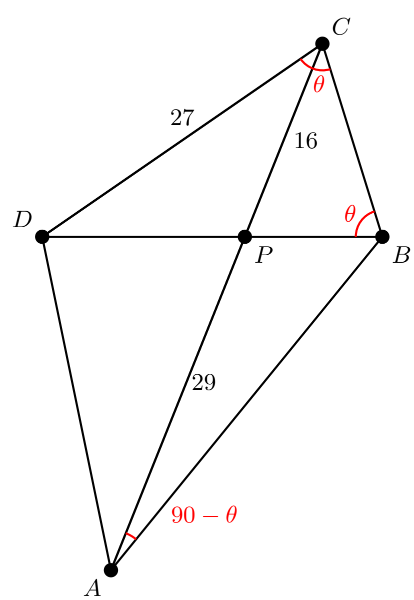
   If the total number of fry bags in Anirudh's completed collection can be written as $a^b - c$, where a, b, c are positive integers, what is the value of $a + b + c$?

10. Aarush rolls 5 standard 6-sided dice. For each of the 5 rolls, he gets a stick of length the number he rolled (Ex. if Aarush rolls a 4, he gets a stick of length 4). Let the probability that Aarush can take 4 of the 5 stickssw and make a rectangle be $\frac{a}{b}$, where a and b are relatively prime numbers. What is $a + b$?

11. Right triangle $\triangle ABC$ with legs $AB = 40$ and $AC = 42$ is drawn with circumcircle ω. The unique circle ξ tangent to $\overline{AB}$ and $\overline{AC}$, as well as internally tangent to ω, is drawn with radius r. Find r^2.

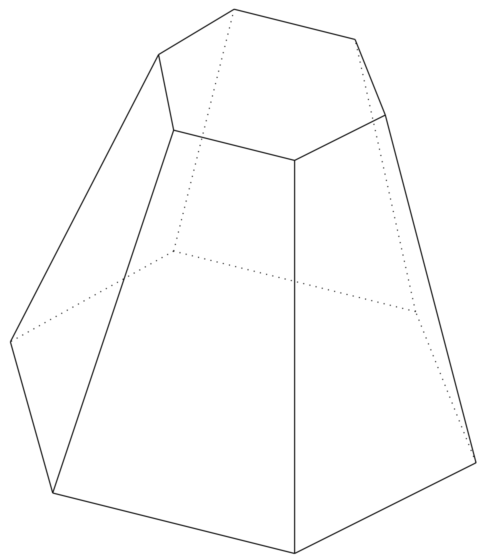
12. Let $f(x)$ be a function such that $\frac{f(x)}{f\left(\frac{2025}{x}\right)} = k_2x^2 + k_1x$ for all $x > 0$. Given that the sum of all possible values of $k_1 + k_2$ can be expressed in the form $\frac{a}{b}$, where a and b are relatively prime positive integers, find $a + b$.

13. Given the following diagram, compute DP^2.



14. The base of Aarush's American Revolution Monument Project is a hexagonal truncated pyramid. Aarush has 5 colors of paint and wants to paint each of the 8 faces such that no two adjacent faces are the same color. In how many ways can he paint the base of his monument?

    (Two faces are considered adjacent if they share an edge. Colorings that differ only by rotations are considered distinct. Refer to the diagram below, but note that it is not drawn to scale.)



15. Triangle ABC has incircle ω and incenter I, and side lengths $AB = 44$, $AC = 52$, and $BC = 68$. Let the feet of the altitudes from I to AB, AC, and BC be D, E, and F, and the midpoints of AB, AC, and BC be P, Q, and R. Let the intersections of DE with PR and QR be A_1 and A_2, the intersections of DF with PQ and QR be B_1 and B_2, and the intersections of EF with PQ and PR be C_1 and C_2. Find $A_1R + A_2R + B_1Q + B_2Q + C_1P + C_2P$.