**Team Name:**________________

## Round 1

1. A *chunk* is a 16 by 16 by 256 block volume, and a *sub-chunk* is a 16 by 16 by 16 cube. How many sub-chunks can fit in a chunk?
2. The interior angles of a pentagon form an increasing arithmetic sequence. Given that the smallest angle has a measure of $24°$, find the measure of the largest angle, in degrees.
3. A uniform wooden cylinder of radius 4 cm and height 23 cm is hollowed out to create a tube of uniform thickness 0.5 cm. The density of wood is $0.75\ \text{g/cm}^3$.

   The ratio between the mass of the tube and the mass of the original cylinder can be expressed as $\frac{a}{b}$, where $a$ and $b$ are relatively prime. Find $a + b$.

1.________________ 2.________________ 3.________________

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## Round 2

1. There are 6 equally spaced points on a circle labeled $A$, $B$, $C$, $D$, $E$, and $F$. How many right triangles can be made by connecting exactly three of these six points?

   **Note:** Congruent triangles in different positions are considered distinct.
2. What is the maximum number of pairwise intersection points between a square, a regular hexagon, and a circle?

   **Note:** Pairwise intersection points are points where two of the three shapes intersect.
3. Eric is bored and starts counting the positive integers: 1, 2, 3, 4, 5, .... However, he really dislikes the number 0, so he skips any number containing zero. (For example, when Eric reaches 99, the next number he counts is 111.)

   When Eric reaches the number 1234, how many numbers has he counted?

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## Round 3

1. Nathan feels like Tanish and Eric should stop fighting over food. Nathan decides to make them work together on a project in exchange for candy. Tanish can finish the project alone in 5 hours, while Eric can finish the project alone in 6 hours.

   Eric starts on the project first. After 2 hours, Eric convinces Tanish to help him by offering him candy, but Eric stops working on the project after 30 minutes of collaboration and leaves Tanish to finish the rest of it.

   If Tanish and Eric spent $t$ and $e$ minutes on the project, respectively, find $t - e$.

2. Tanish realized that Nathan wasn't paying him enough candy. He demands for Nathan to pay him 350 jolly ranchers per hour and an extra 200 jolly ranchers as a base fee. Instead, Nathan offers him 250 skittle packs per hour and an extra 50 skittle packs as a base fee.

   Tanish decides that a skittle pack is worth 1.5 times as much as a jolly rancher. What is the minimum number of hours Tanish needs to work in order for Nathan's offer to be as or more rewarding?

3. After this candy fight, Tanish tries to draw the Deathly Hallows, a symbol from his favorite book series, Harry Potter. It consists of a circle $\omega$ of radius 5 inscribed in an equilateral triangle. One altitude of the triangle with height $h$ is drawn.

   Tanish begins by drawing $\omega$. However, he accidentally inscribes the circle in an acute isosceles triangle whose longest altitude is $h + 5$. The area of this isosceles triangle can be expressed as $m\sqrt{n}$, where $n$ is not divisible by the square of any prime. Find $m + n$.

1.________________ 2.________________ 3.________________

**Team Name:**________________

## Round 4

1. The roots of the polynomial $f(x) = 2x^4 + 5x^3 - 16x^2 + 11x + 7$ are $w, x, y,$ and $z$. Compute the value of

$$-w^2x^2yz - w^2xy^2z - w^2xyz^2 - wx^2y^2z - wx^2yz^2 - wxy^2z^2.$$

2. Let $\triangle ABC$ be an isosceles triangle such that $AB = AC = 2$ and $\angle ABC = 36^\circ$. The length of $\overline{BC}$ can be expressed in the form $a + \sqrt{b}$, where $a$ and $b$ are integers. Find $a + b$.

3. Ryan is creating a sequence of numbers. First, Ryan chooses a positive integer from 1 to 999 (inclusive). Then, he follows three rules to create the next number:

   (a) If the number is prime, multiply it by the next positive integer.

   (b) If the number is less than 20 and not prime, cube it.

   (c) Otherwise, divide it by its smallest prime factor.

   Ryan repeats this until he gets a number greater than or equal to 1000. Given there are a total of 168 prime numbers less than 1000, how many possible numbers can Ryan end with?

1.________________ 2.________________ 3.________________

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## Round 5

1. Triangle $\triangle ABC$ with $AB = 13$, $BC = 14$, $AC = 15$ is inscribed in a circle $\omega$. Circles $\omega_1$, $\omega_2$, $\omega_3$ are internally tangent to $\omega$ and externally tangent to $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ respectively. The maximum product of the areas of $\omega_1$, $\omega_2$, and $\omega_3$ can be expressed as $\frac{m}{n} \cdot \pi^3$, where $m$ and $n$ are relatively prime. Find $m + n$.

2. A car is driving recklessly on a five-lane highway. Every minute, the car will either stay on the same lane, switch to the lane directly to its left, or switch to the lane directly to its right, all with a 1/3rd probability. If the car switches left while on the leftmost lane or right while on the rightmost lane, it will crash.

   If the car starts on the middle lane, it will drive for an expected time of $m$ minutes before crashing. If the car starts on the leftmost lane, it will drive for an expected time of $l$ minutes before crashing. Find $m + l$.

3. The *Lucas sequence* $\ell_n$ is defined on the positive integers by $\ell_1 = 2$, $\ell_2 = 1$, and $\ell_n = \ell_{n-1} + \ell_{n-2}$ for $n \geq 3$. The first few terms of the Lucas sequence are $\{2, 1, 3, 4, 7, 11, ...\}$

   Now, consider the following infinite series, where the $n$th term is the $n$th Lucas number divided by $4^n$.

$$\frac{2}{4} + \frac{1}{16} + \frac{3}{64} + \frac{4}{256} + \cdots + \frac{\ell_n}{4^n} + \cdots.$$

   This sum approaches a value denoted $\frac{a}{b}$, where $a$ and $b$ are relatively prime. Find $a + b$.

1.________________ 2.________________ 3.________________

**Team Name:**________________

## Round 6

1. A number $N$ with distinct and nonzero digits is called *interesting* if $N$ is divisible by all of its digits. What is the largest prime factor of the largest *interesting* number?

2. Given $x + \frac{1}{x} = 1$, find the absolute value of $\frac{1}{x^{2025}} + \frac{1}{x^{2024}} + x^{2024} + x^{2025}$.

3. Derek, Derrick, Eric, Erik, and Frederick are playing a game. Derek and Derrick have a $\frac{1}{6}$ chance of winning the game on their turns. Eric and Erik have a $\frac{1}{3}$ chance of winning the game on their turns. Frederick has a $\frac{19}{100}$ chance of winning the game on his turn. Turns are determined alphabetically by first name, with Derek going first.

   The probabilities that the winner of the game has a name starting with "D", "E", and "F" are $p_d$, $p_e$, and $p_f$, respectively. The ratio $p_d : p_e : p_f$ can be expressed as $d : e : f$, where $d$, $e$, and $f$ are relatively prime positive integers. Find $d + e - f$.

1.________________ 2.________________ 3.________________

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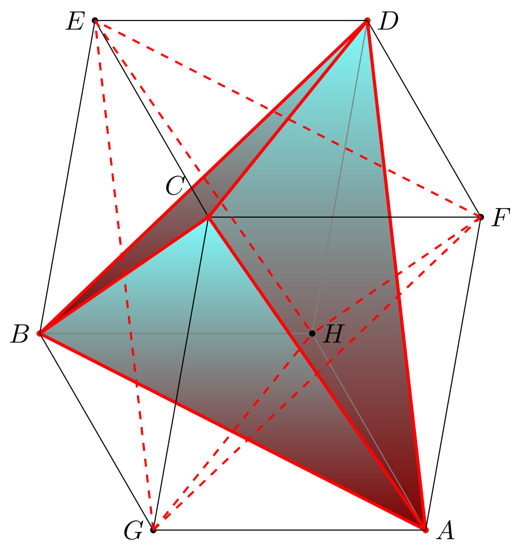
**Round 7**

1. Let $f(x)$ be a polynomial of degree 2025 and leading coefficient 2026. For $k = 1, 2, 3, ..., 2024$, the relation $f(k) = 3k + 7$ holds true. Compute $f(2025) - f(0) - 2026!$.

2. Below is a diagram of a tetrahedron inscribed in a cube of side length 12. Compute the volume of the intersection of the two tetrahedra $ABCD$ and $EFGH$.




3. Let $f(x) = \frac{3}{8}x - \frac{5}{4}$. Define $f^n(x)$ recursively by $f^n(x) = f^{n-1}(f(x))$ and $f^1(x) = f(x)$. Let $m$ be the smallest positive integer such that $f^7(m)$ an integer.

   The prime factorization of $m$ can be written as $m = p_1^{e_1} \cdot p_2^{e_2} \cdot p_3^{e_3} \cdot p_4^{e_4} \cdot p_5^{e_5} \cdot p_6^{e_6}$, where $p_1, p_2, \ldots, p_6$ are distinct primes, and $e_1, e_2, \ldots, e_6$ are their corresponding exponents.

   Find the positive difference between the sum of the primes and the sum of their exponents.

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**Estimation**

Find the positive integer closest to the following value $V$:

$$V = \sqrt[2025]{2025!} = \sqrt[2025]{1 \cdot 2 \cdot 3 \cdot 4 \cdots 2025}.$$

Your answer will be scored according to a modified bell curve. In particular, if your input is $I$ and the correct answer is $X$, your score for this round will be

$$13 - 26 \left| 0.5 - \text{normcdf}(\text{input } I, \text{ mean } X^{1/3}, \text{ stdev } X^{1/3}) \right|.$$

Answer:________________