Name: ______________________ **Score:** _______/50 **Grade:** _____

The accuracy round is 40 minutes long. Questions are sorted and weighted by difficulty.

1. ____________ (3pt)
2. ____________ (3pt)
3. ____________ (4pt)
4. ____________ (4pt)
5. ____________ (5pt)
6. ____________ (5pt)
7. ____________ (6pt)
8. ____________ (6pt)
9. ____________ (7pt)
10. ____________ (7pt)
11. ____________ (Tiebreaker)

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1. What is $(20 \cdot 25) \cdot (20 + 25)$?

2. Amy is bored, so she starts spinning pencils. She performs one round of tricks per minute. She will drop the pencil on the first trick with probability $\frac{1}{5}$, the second trick with probability $\frac{2}{5}$, and the third trick with probability $\frac{4}{5}$.

   Once Amy drops the pencil five times, she will stop spinning after finishing the entire round of tricks. The expected number of minutes Amy will last spinning pencils is $\frac{a}{b}$, where a and b are relatively prime. Find $a+b$.

3. In Speed #5, Eric received the least amount of cookies from Tanish. To take revenge on him, Eric will throw a pizza party without him. Unfortunately, he has invited 4 of Tanish's acquaintances and 2 of his friends. Each acquaintance has a $\frac{1}{6}$ chance of telling Tanish about the party, and each friend has an $\frac{4}{5}$ chance of telling Tanish.

   If no one else at the party knows Tanish, the probability that Tanish finds out about Eric's pizza party can be expressed as $\frac{a}{b}$, where a and b are relatively prime. Find $a + b$.

4. In circle $\odot O$, points C and D divide chord $\overline{AB}$ into three equal segments, each of length 6. If the lengths CO and DO are also both equal to 6, the radius of the circle can be expressed as $a\sqrt{b}$, where b is not divisible by the square of any prime. Find $a + b$.

5. Anirudh the Astronaut landed his rocket on another planet. The aliens of the planet, however, don't use base 10 like humans do; they use an unknown integer base b. The aliens' lucky number, z, is a trendy topic on this planet.

   One alien says, "The cube of our lucky number is 247 (base b)."

   Another adds, "The value b in base z is 15."

   What is $z + b$ (in base 10)?

6. In rectangle $WXYZ$, points A and B lie on $\overline{WX}$ such that $WA = AB = BX$, and point C is the midpoint of $\overline{YZ}$. $\overline{XZ}$ intersects $\overline{AC}$ and $\overline{BC}$ at points D and E, respectively. The ratio of the areas of $BEX : ABED : DEC$ can be expressed as $a : b : c$, where a, b, and c are relatively prime positive integers. Find $a + b + c$.

7. How many 4-digit numbers $\underline{abcd}$ are there such that $a \le b \le c \le d \le 6$ and $a \ne 0$?

   Note: The notation $\underline{abcd}$ denotes the number $1000a + 100b + 10c + d$, not the product $a \times b \times c \times d$.

8. This following infinite series

   $$\frac{1}{4^2} + \frac{1}{5^2+2} + \frac{1}{6^2+4} + \frac{1}{7^2+6} + \frac{1}{8^2+8} + \cdots + \frac{1}{(n+3)^2 + 2(n-1)} + \cdots$$

   approaches a value denoted $\frac{a}{b}$, where a and b are relatively prime. Find $a + b$.

9. The number 1337 can be split into a sum of positive integers in many ways.

   $$1337 = (668 + 669) = (49 + 51 + 127 + 553 + 557) = (1 + 2 + \underbrace{4 + 4 + \cdots + 4}_{332 \text{ fours}} + 6)$$

   Formally, define a 1337-*partition* $\mathcal{P}$ to be a n-tuple $(n_1, n_2, n_3, \ldots, n_k)$ of nondecreasing positive integers such that $n_1 + n_2 + n_3 + \cdots + n_k = 1337$. Then, define a function $f(\mathcal{P})$ which multiplies every term in the partition together. For example, $f(668, 669) = 668 \times 669 = 446892$ and $f(49, 51, 127, 553, 557) = 49 \times 51 \times 127 \times 553 \times 557 = 97757548833$.

   The maximum value of $f(\mathcal{P})$ is achieved with a partition $\mathcal{P}^\star$ with t terms. Find t.

10. Suppose $x_n = 15^n - 7^n$. Find the remainder when $x_{2025} - x_{2024}$ is divided by 121.

11. **Estimation:** Consider the harmonic series h_n defined on the positive integers, which diverges as n increases boundlessly:

    $$h_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} + \ldots$$

    The smallest integer N such that $h_N > 2025$ has d digits. Find d.

    (This question serves as a **tiebreaker** for the individual rounds.)