Acton-Boxborough Math Competition November Contest

Saturday, November 20 — Sunday, November 21, 2021

Contest Rules and Format

The 2021 November Contest consists of 15 problems — each with an answer between 0 and 100,000. The contest window is

Saturday, November 20 to Sunday, November 21.

The following rules supersede any others regarding the Online Contests.

- To submit your responses, fill out a form on our website, `abmathcompetition.org`. Include your email, name, school, grade (we require this information for prize purposes).
- Submissions will close at 10:00PM Eastern Standard Time, on the final day of the round.
- Computational aids such as calculators, abaci, slide rules, etc. are prohibited. Drawing aids such as protractors and rules are permissible, but computer software such as GeoGebra or Desmos are prohibited.
- You may only work individually on the problems—consultation with others is not permitted.
- You may take as much time as you wish during the contest window.
- Each problem is worth 1 point.
- Ties will be broken by the "most difficult" problem solved. If problem A is solved by a contestants, and problem B is solved by b contestants, with $a < b$, then problem A is more difficult than B.

Awards and Prizes

- Top scoring contestants within each round will have their names posted on our website.
- Scores will be emailed to contestants within one week of the contest's end.
- Top scoring contestants across all three rounds will receive prizes.
- Physical prizes (such as AoPS gift certificates, merchandise, calculators, etc.) will be given to top scorers if they attend the ABMC onsite contest.

Thanks to our Sponsors!

Good luck!

Problems

1. Martin's car insurance costed \$6000 before he switched to Geico, when he saved 15% on car insurance. When Mayhem switched to Allstate, he, a safe driver, saved 40% on car insurance. If Mayhem and Martin are now paying the same amount for car insurance, how much was Mayhem paying before he switched to Allstate?

2. The 7-digit number N can be written as $\underline{A}\ \underline{2}\ \underline{0}\ \underline{B}\ \underline{2}\ \underline{1}\ \underline{5}$. How many values of N are divisible by 9?

3. The solutions to the equation $x^2 - 18x - 115 = 0$ can be represented as a and b. What is $a^2 + 2ab + b^2$?

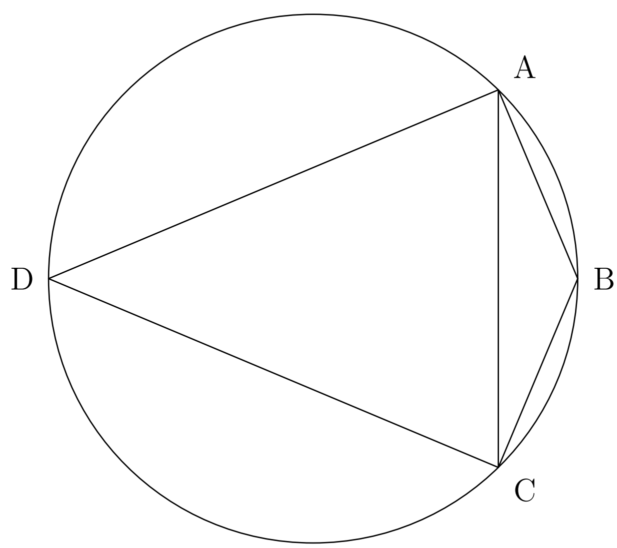
4. The exterior angles of a regular polygon measure to 4 degrees. What is a third of the number of sides of this polygon?

5. Charlie Brown is having a thanksgiving party.

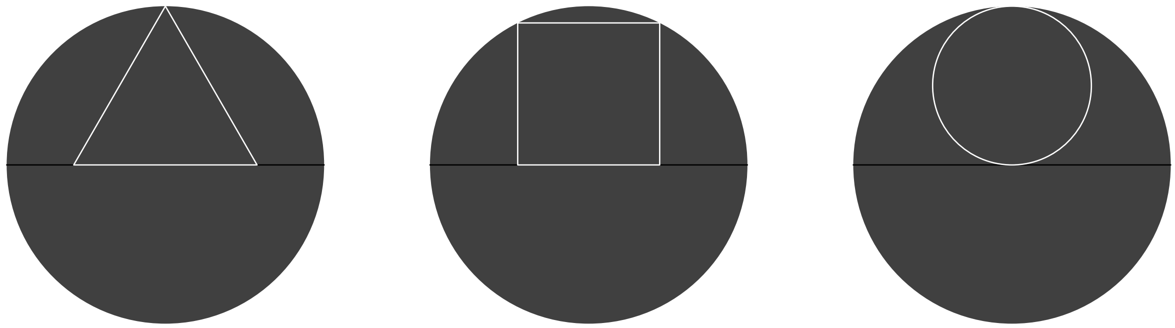
   - He wants one turkey, with three different sizes to choose from.
   - He wants to have two or three vegetable dishes, when he can pick from Mashed Potatoes, Sautéed Brussels Sprouts, Roasted Butternut Squash, Buttery Green Beans, and Sweet Yams;
   - He wants two desserts out of Pumpkin Pie, Apple Pie, Carrot Cake, and Cheesecake.

   How many different combinations of menus are there?

6. In the diagram below, $\overline{AD} \cong \overline{CD}$ and $\triangle DAB$ is a right triangle with $\angle DAB = 90°$. Given that the radius of the circle is 6 and $m\angle ADC = 30°$, if the length of minor arc $\widehat{AB}$ is written as $a\pi$, what is a?



7. This Halloween, Owen and his two friends dressed up as guards from Squid Game. They needed to make three masks, which were black circles with a white equilateral triangle, circle, or square inscribed in their upper halves. Resourcefully, they used black paper circles with a radius of 5 inches and white tape to create these masks. Ignoring the width of the tape, how much tape did they use? If the length can be expressed $a\sqrt{b} + c\sqrt{d} + \frac{e}{f} \cdot \pi$ such that b and d are not divisible by the square of any prime, and e and f are relatively prime, find $a + b + c + d + e + f$.



8. Given $LCM(10^8, 8^{10}, n) = 20^{15}$, where n is a positive integer, find the total number of possible values of n.

9. If one can represent the infinite progression $\frac{1}{11} + \frac{2}{13} + \frac{3}{121} + \frac{4}{169} + \frac{5}{1331} + \frac{6}{2197} \ldots$ as $\frac{a}{b}$, where a and b are relatively prime positive integers, what is a?

10. Consider a tiled 3x3 square without a center tile. How many ways are there to color the squares such that no two colored squares are adjacent (vertically or horizontally)? Consider rotations of an configuration to be the same, and consider the no-color configuration to be a coloring.

11. Let ABC be a triangle with $AB = 4$ and $AC = 7$. Let AD be an angle bisector of triangle ABC. Point M is on AC such that AD intersects BM at point P, and $AP : PD = 3 : 1$. If the ratio $AM : MC$ can be expressed as $\frac{a}{b}$ such that a, b are relatively prime positive integers, find $a + b$.

12. For a positive integer n, define $f(n)$ as the number of positive integers less than or equal to n that are coprime with n. For example, $f(9) = 6$ because 9 does not have any common divisors with 1, 2, 4, 5, 7, or 8.

    Calculate:

$$\sum_{i=2}^{100} \left(29^{f(i)} \mod i \right).$$

13. Let ABC be an equilateral triangle. Let P be a randomly selected point in the incircle of ABC. Find $a+b+c+d$ if the probability that $\angle BPC$ is acute can be expressed as $\frac{a\sqrt{b} - c\pi}{d\pi}$ for positive integers a, b, c, d where $\gcd(a, c, d) = 1$ and b is not divisible by the square of any prime.

14. When the following expression is simplified by expanding then combining like terms, how many terms are in the resulting expression?

$$(a + b + c + d)^{100} + (a + b - c - d)^{100}$$

15. Jerry has a rectangular box with integral side lengths. If 3 units are added to each side of the box, the volume of the box is tripled. What is the largest possible volume of this box?