Acton-Boxborough Math Competition Online Contest

Saturday, October 17 — Sunday, October 18, 2020

Contest Rules and Format

The 2020 October Contest consists of 15 problems — each with an answer between 0 and 100,000. The contest window is

Saturday, October 17 to Sunday, October 18.

The following rules supersede any others regarding the Online Contests.

- To submit your responses, fill out a form on our website, `abmathcompetition.org`. Include your email, name, school, grade (we require this information for prize purposes).
- Submissions will close at 10:00PM Eastern Standard Time, on the final day of the round.
- Computational aids such as calculators, abaci, slide rules, etc. are prohibited. Drawing aids such as protractors and rules are permissible, but computer software such as GeoGebra or Desmos are prohibited.
- You may only work individually on the problems—consultation with others is not permitted.
- You may take as much time as you wish during the contest window.
- Each problem is worth 1 point.
- Ties will be broken by the "most difficult" problem solved. If problem A is solved by a contestants, and problem B is solved by b contestants, with $a < b$, then problem A is more difficult than B.

Awards and Prizes

- Top scoring contestants within each round will have their names posted on our website.
- Scores will be emailed to contestants within one week of the contest's end.
- Top scoring contestants across all three rounds will receive prizes.
- Physical prizes (such as AoPS gift certificates, merchandise, calculators, etc.) will be given to top scorers if they attend the ABMC onsite contest.

Thanks to our Sponsors!

Good luck!

Problems

1. Catherine's teacher thinks of a number and asks her to subtract 5 and then multiply the result by 6. Catherine accidentally switches the numbers by subtracting 6 and multiplying by 5 to get 30. If Catherine had not swapped the numbers, what would the correct answer be?

2. At Acton Boxborough Regional High School, desks are arranged in a rectangular grid-like configuration. In order to maintain proper social distancing, desks are required to be at least 6 feet away from all other desks. Assuming that the size of the desks is negligible, what is the maximum number of desks that can fit in a 25 feet by 25 feet classroom?

3. Joshua hates writing essays for homework, but his teacher Mr. Meesh assigns two essays every 3 weeks. However, Mr. Meesh favors Joshua, so he allows Joshua to skip one essay out of every 4 that are assigned. How many essays does Joshua have to write in a 24-week school year?

4. Libra likes to read, but she is easily distracted. If a page number is even, she reads the page twice. If a page number is an odd multiple of three, she skips it. Otherwise, she reads the page exactly once. If Libra's book is 405 pages long, how many pages in total does she read if she starts on page 1? (Reading the same page twice counts as two pages.)

5. Let the GDP of an integer be its ***G***reatest ***D***ivisor that is ***P***rime. For example, the GDP of 14 is 7. Find the largest integer less than 100 that has a GDP of 3.

6. As has been proven by countless scientific papers, the Earth is a flat circle. Bob stands at a point on the Earth such that if he walks in a straight line, the maximum possible distance he can travel before he falls off is 7 miles, and the minimum possible distance he can travel before he falls off is 3 miles. Then the Earth's area in square miles is $k\pi$ for some integer k. Compute k.

7. Edward has 2 magical eggs. Every minute, each magical egg that Edward has will double itself. But there's a catch. At the end of every minute, Edward's brother Eliot will come outside and smash one egg on his forehead, causing Edward to lose that egg permanently. For example, starting with 2 eggs, after one minute there will be 3 eggs, then 5, 9, and so on. After 1 hour, the number of eggs can be expressed as $a^b + c$ for positive integers a, b, c where $a > 1$, and a and c are as small as possible. Find $a + b + c$.

8. Define a sequence of real numbers $a_1, a_2, a_3, \ldots, a_{2019}, a_{2020}$ with the property that $a_n = \dfrac{a_{n-1} + a_n + a_{n+1}}{3}$ for all $n = 2, 3, 4, 5, \cdots 2018, 2019$. Given that $a_1 = 1$ and $a_{1000} = 1999$, find a_{2020}.

9. In $\triangle ABC$ with $AB = 10$ and $AC = 12$, points D and E lie on sides $\overline{AB}$ and $\overline{AC}$, respectively, such that $AD = 4$ and $AE = 5$. If the area of quadrilateral BCED is 40, find the area of $\triangle ADE$.

10. A positive integer is called *powerful* if every prime in its prime factorization is raised to a power greater than or equal to 2. How many positive integers less than 100 are powerful?

11. Let integers $A, B < 10,000$ be the populations of Acton and Boxborough, respectively. When A is divided by B, the remainder is 1. When B is divided by A, the remainder is 2020. If the sum of the digits of A is 17, find the total combined population of Acton and Boxborough.

12. Let $a_1, a_2, \cdots, a_n$ be an increasing arithmetic sequence of positive integers. Given $a_n - a_1 = 20$ and $a_n^2 - a_{n-1}^2 = 63$, find the sum of the terms in the arithmetic sequence.

13. Bob rolls a cubical, an octahedral and a dodecahedral die $(6, 8$ and 12 sides respectively) numbered with the integers from 1 to 6, 1 to 8 and 1 to 12 respectively. If the probability that the sum of the numbers on the cubical and octahedral dice equals the number on the dodecahedral die can be written as $\frac{m}{n}$, where m, n are relatively prime positive integers, compute $n - m$.

14. Let $\triangle ABC$ be inscribed in a circle with center O with $AB = 13, BC = 14, AC = 15$. Let the foot of the perpendicular from A to BC be D and let AO intersect BC at E. Given the length of DE can be expressed as $\dfrac{m}{n}$ where m, n are relatively prime positive integers, find $m + n$.

15. The set S consists of the first 10 positive integers. A collection of 10 not necessarily distinct integers is chosen from S at random. If a particular number is chosen more than once, all but one of its occurrences are removed. Call the set of remaining numbers A. Let $\frac{a}{b}$ be the expected value of the number of the elements in A, where a, b are relatively prime positive integers. Find the reminder when $a + b$ is divided by 1000.