Acton-Boxborough Math Competition November Contest

Saturday, November 21 — Sunday, November 22, 2020

Contest Rules and Format

The 2020 November Contest consists of 15 problems — each with an answer between 0 and 100,000. The contest window is

Saturday, November 21 to Sunday, November 22.

The following rules supersede any others regarding the Online Contests.

- To submit your responses, fill out a form on our website, `abmathcompetition.org`. Include your email, name, school, grade (we require this information for prize purposes).
- Submissions will close at 10:00PM Eastern Standard Time, on the final day of the round.
- Computational aids such as calculators, abaci, slide rules, etc. are prohibited. Drawing aids such as protractors and rules are permissible, but computer software such as GeoGebra or Desmos are prohibited.
- You may only work individually on the problems—consultation with others is not permitted.
- You may take as much time as you wish during the contest window.
- Each problem is worth 1 point.
- Ties will be broken by the "most difficult" problem solved. If problem A is solved by a contestants, and problem B is solved by b contestants, with $a < b$, then problem A is more difficult than B.

Awards and Prizes

- Top scoring contestants within each round will have their names posted on our website.
- Scores will be emailed to contestants within one week of the contest's end.
- Top scoring contestants across all three rounds will receive prizes.
- Physical prizes (such as AoPS gift certificates, merchandise, calculators, etc.) will be given to top scorers if they attend the ABMC onsite contest.

Thanks to our Sponsors!

Good luck!

Problems

1. A large square is cut into four smaller, congruent squares. If each of the smaller squares has perimeter 4, what was the perimeter of the original square?

2. Pie loves to bake apples so much that he spends 24 hours a day baking them. If Pie bakes a dozen apples in one day, how many minutes does it take Pie to bake one apple, on average?

3. Bames Jond is sent to spy on James Pond. One day, Bames sees James type in his 4-digit phone password. Bames remembers that James used the digits $0, 5$, and 9, and no other digits, but he does not remember the order. How many possible phone passwords satisfy this condition?

4. What do you get if you square the answer to this question, add 256 to it, and then divide by 32?

5. Chloe the Horse and Flower the Chicken are best friends. When Chloe gets sad for any reason, she calls Flower, so Chloe must remember Flower's 3 digit phone number, which can consist of any digits 0-5. Given that the phone number's digits are unique and add to 5, the number does not start with 0, and the 3 digit number is prime, what is the sum of all possible phone numbers?

6. Anuj has a circular pizza with diameter A inches, which is cut into B congruent slices, where A, B are positive integers. If one of Anuj's pizza slices has a perimeter of $3\pi + 30$ inches, find $A + B$.

7. Bob really likes to study math. Unfortunately, he gets easily distracted by messages sent by friends. At the beginning of every minute, there is an $\frac{6}{10}$ chance that he will get a message from a friend. If Bob does get a message from a friend, there is a $\frac{9}{10}$ chance that he will look at the message, causing him to waste 30 seconds before resuming his studying. If Bob doesn't get a message from a friend, there is a $\frac{3}{10}$ chance Bob will still check his messages hoping for a message from his friends, wasting 10 seconds before he resumes his studying. What is the expected number of minutes in 100 minutes for which Bob will be studying math?

8. Suppose there is a positive integer n with 225 distinct positive integer divisors. What is the minimum possible number of divisors of n that are perfect squares?

9. Let a, b, c be positive integers. a has 12 divisors, b has 8 divisors, c has 6 divisors, and $\text{lcm}(a, b, c) = abc$. Let d be the number of divisors of a^2bc. Find the sum of all possible values of d.

10. Let $\triangle ABC$ be a triangle with side lengths $AB = 17, BC = 28, AC = 25$. Let the altitude from A to BC and the angle bisector of angle B meet at P. Given the length of BP can be expressed as $\frac{a\sqrt{b}}{c}$ for positive integers a, b, c where $\gcd(a, c) = 1$ and b is not divisible by the square of any prime, find $a + b + c$.

11. Let a, b, and c be the roots of the cubic equation $x^3 - 5x + 3 = 0$. Let $S = a^4b + ab^4 + a^4c + ac^4 + b^4c + bc^4$. Find $|S|$.

12. Call a number *palindromeish* if changing a single digit of the number into a different digit results in a new six-digit palindrome. For example, the number 110012 is a *palindromeish* number since you can change the last digit into a 1, which results in the palindrome 110011. Find the number of 6 digit *palindromeish* numbers.

13. Let $P(x)$ be a polynomial of degree 3 with real coefficients and leading coefficient 1. Let the roots of $P(x)$ be a, b, c. Given that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 4$ and $a^2 + b^2 + c^2 = 36$, the coefficient of x^2 is negative, and $P(1) = 2$, let the S be the sum of possible values of $P(0)$. Then $|S|$ can be expressed as $\frac{a + b\sqrt{c}}{d}$ for positive integers a, b, c, d such that $\gcd(a, b, d) = 1$ and c is not divisible by the square of any prime. Find $a + b + c + d$.

14. Let ABC be a triangle with side lengths $AB = 7, BC = 8, AC = 9$. Draw a circle tangent to AB at B and passing through C. Let the center of the circle be O. The length of AO can be expressed as $\frac{a\sqrt{b}}{c\sqrt{d}}$ for positive integers a, b, c, d where $\gcd(a, c) = \gcd(b, d) = 1$ and b, d are not divisible by the square of any prime. Find $a + b + c + d$.

15. Many students in Mr. Noeth's BC Calculus class missed their first test, and to avoid taking a makeup, have decided to never leave their houses again. As a result, Mr. Noeth decides that he will have to visit their houses to deliver the makeup tests. Conveniently, the 17 absent students in his class live in consecutive houses on the same street. Mr. Noeth chooses at least three of every four people in consecutive houses to take a makeup. How many ways can Mr. Noeth select students to take makeups?