Team Name:________________

Round 1

1. A classroom has 29 students. A teacher needs to split up the students into groups of at most 4. What is the minimum number of groups needed?
2. On his history map quiz, Eric recalls that Sweden, Norway and Finland are adjacent countries, but he has forgotten which is which, so he labels them in random order. The probability that he labels all three countries correctly can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.
3. In a class of 40 sixth graders, the class average for their final test comes out to be 90 (out of a 100). However, a student brings up an issue with problem 5, and 10 students receive credit for this question, bringing the class average to a 90.75. How many points was problem 5 worth?

1.________________ 2.________________ 3.________________

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Round 2

1. Compute $1 - 2 + 3 - 4 + \cdots - 2022 + 2023$.
2. In triangle ABC, $\angle ABC = 75°$. Point D lies on side AC such that $BD = CD$ and $\angle BDC$ is a right angle. Compute the measure of $\angle A$.
3. Joe is rolling three four-sided dice each labeled with positive integers from 1 to 4. The probability the sum of the numbers on the top faces of the dice is 6 can be written as $\frac{p}{q}$ where p and q are relatively prime integers. Find $p + q$.

1.________________ 2.________________ 3.________________

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Round 3

1. For positive integers a, b, c, d that satisfy $a+b+c+d=23$, what is the maximum value of $abcd$?

2. A buckball league has twenty teams. Each of the twenty teams plays exactly five games with each of the other teams. If each game takes 1 hour and thirty minutes, then how many total hours are spent playing games?

3. For a triangle $\triangle ABC$, let M, N, O be the midpoints of AB, BC, AC, respectively. Let P, Q, R be points on AB, BC, AC such that $AP = \frac{1}{3}AB, BQ = \frac{1}{3}BC$, and $CR = \frac{1}{3}AC$. The ratio of the areas of $\triangle MNO$ and $\triangle PQR$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

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Round 4

1. 2023 has the special property that leaves a remainder of 1 when divided by 2, 21 when divided by 22, and 22 when divided by 23. Let n equal the lowest integer greater than 2023 with the above properties. What is n?

2. Ants A, B are on points $(0,0)$ and $(3,3)$ respectively, and ant A is trying to get to $(3,3)$ while ant B is trying to get to $(0,0)$. Every second, ant A will either move up or right one with equal probability, and ant B will move down or left one with equal probability. The probability that the ants will meet each other be $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a+b$.

3. Find the number of trailing zeros of $100!$ in base 49.

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Round 5

1. In a triangle $\triangle ABC$ with $AB = 48$, let the angle bisectors of $\angle BAC$ and $\angle BCA$ meet at I. Given $\frac{[ABI]}{[BCI]} = \frac{24}{7}$ and $\frac{[ACI]}{[ABI]} = \frac{25}{24}$, find the area of $\triangle ABC$.

2. At a dinner party, 9 people are to be seated at a round table. If person A cannot be seated next to person B and person C cannot be next to person D, how many ways can the 9 people be seated? Rotations of the table are indistinguishable.

3. Let $f(x)$ be a monic cubic polynomial such that $f(1) = f(7) = f(10) = a$ and $f(2) = f(5) = f(11) = b$. Find $|a - b|$.

1.________________ 2.________________ 3.________________

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Round 6

1. If N has 16 positive integer divisors and the sum of all divisors of N that are multiples of 3 is 39 times the sum of divisors of N that are not multiples of 3, what is the smallest value of N?

2. In the two parabolas $y = x^2/16$ and $x = y^2/16$, the single line tangent to both parabolas intersects the parabolas at A and B. If the parabolas intersect each other at C which is not the origin, find the area of $\triangle ABC$.

3. Five distinguishable noncollinear points are drawn. How many ways are there to draw segments connecting the points, such that there are exactly two disjoint groups of connected points? Note that a single point can be considered a connected group of points.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 7

1. Let a, b be positive integers, and $1 = d_1 < d_2 < d_3 < \cdots < d_n = a$ be the divisors of a, and $1 = e_1 < e_2 < e_3 < \cdots < e_m = b$ be the divisors of b. Given $\gcd(a, b) = d_2 = e_6$, find the smallest possible value of $a + b$.

2. Let $\triangle ABC$ be a triangle such that $AB = 2$ and $AC = 3$. Let X be the point on BC such that $m\angle BAX = \frac{1}{3} m\angle BAC$. Given that $AX = 1$, the sum of all possible values of CX^2 can be expressed as $\frac{a}{b}$ for relatively prime positive integers a, b. Find $a + b$.

3. Bob has a playlist of 6 different songs in some order, and he listens to his playlist repeatedly. Every time he finishes listening to the third song in the playlist, he randomly shuffles his playlist and listens to the playlist starting with the new first song. The expected number of times Bob shuffles his songs before he listens each one of his 6 songs at least once can be expressed as $\frac{a}{b}$ for relatively prime positive integers a and b. Find $a+b$.

1.________________ 2.________________ 3.________________

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Round 8

1. $\underline{A}, \underline{B}, \underline{C}, \underline{D}, \underline{E}, \underline{F}, \underline{G}, \underline{H}, \underline{I}$, and $\underline{J}$ represent distinct digits (0 to 9) in the equation $\underline{FBGA} - \underline{ABAC} = \underline{DCE}$ (where $\underline{ABAC}$ and $\underline{FBGA}$ are four-digit numbers, and $\underline{DCE}$ is a three-digit number). If $\underline{A} < \underline{B} < \underline{C} < \underline{D}$ and $\underline{ABCDEFGHIJ}$ is minimized, find $\underline{ABCD} + \underline{EFG} + \underline{HI} + \underline{J}$.

2. $\underline{A}, \underline{B}, \underline{C}, \underline{D}, \underline{E}$, and $\underline{F}$ represent distinct digits (0 to 9) in the equations $\underline{ABC} \cdot \underline{C} = \underline{DEA}$, $\underline{ABC} \cdot \underline{D} = \underline{BAFE}$, and $\underline{DEA} + \underline{BAFE}0 = \underline{BFACA}$ (where $\underline{ABC}$ and $\underline{DEA}$ are three-digit numbers, $\underline{BAFE}$ is a four-digit number, and $\underline{BFACA}$ is a five-digit number). Find $\underline{ABC} + \underline{DE} + \underline{F}$.

3. $\underline{A}, \underline{B}, \underline{C}, \underline{D}, \underline{E}, \underline{F}, \underline{G}$, and $\underline{H}$ represent distinct digits (0 to 9) in the equations $\underline{ABC} \cdot \underline{D} = \underline{AFGE}$, $\underline{ABC} \cdot \underline{C} = \underline{GHC}$, $\underline{GHC} + \underline{HFF} = \underline{AEHC}$, and $\underline{AFGE}0 + \underline{AEHC} = \underline{AEABC}$ (where $\underline{ABC}, \underline{GHC}$ and $\underline{HFF}$ are three-digit numbers, $\underline{AFGE}$ is a four-digit number, and $\underline{AEABC}$ is a five-digit number). Find $\underline{ABCD} + \underline{EFGH}$.

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Round 9

Estimate the arithmetic mean of all answers to this question. Only integer answers between 0 to $100,000$ will count for credit and count toward the average.

Your answer will be scored according to the following formula, where X is the correct answer and I is your input.

$$\max\left\{0, \left\lceil \min\left\{13 - \frac{|I-X|}{0.05|I|}, 13 - \frac{|I-X|}{0.05|I-2X|}\right\}\right\rceil\right\}.$$

Answer:____________