Name: ________________ **Score:** ______/______/50 **Grade:** ____

The speed round is 30 minutes long. Questions are weighted by difficulty.

1. ______	6. ______	11. ______	16. ______	21. ______
2. ______	7. ______	12. ______	17. ______	22. ______
3. ______	8. ______	13. ______	18. ______	23. ______
4. ______	9. ______	14. ______	19. ______	24. ______
5. ______	10. ______	15. ______	20. ______	25. ______

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1. Compute $2^2 + 0 \cdot 0 + 2^2 + 3^3$.

2. How many total letters (not necessarily distinct) are there in the names Jerry, Justin, Jackie, Jason, and Jeffrey?

3. What is the remainder when 20232023 is divided by 50?

4. Let $ABCD$ be a square. The fraction of the area of $ABCD$ that is the area of the intersection of triangles ABD and ABC can be expressed as $\frac{a}{b}$, where a and b relatively prime positive integers. Find $a + b$.

5. Raymond is playing basketball. He makes a total of 15 shots, all of which are either worth 2 or 3 points. Given he scored a total of 40 points, how many 2-point shots did he make?

6. If a fair coin is flipped 4 times, the probability that it lands on heads more often than tails is $\frac{a}{b}$, where a and b relatively prime positive integers. Find $a + b$.

7. What is the sum of the perfect square divisors of 640?

8. A regular hexagon and an equilateral triangle have the same perimeter. The ratio of the area between the hexagon and equilateral triangle can be expressed in the form $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.

9. If a cylinder has volume 1024π, radius of r and height h, how many ordered pairs of integers (r, h) are possible?

10. Pump A can fill up a balloon in 3 hours, while pump B can fill up a balloon in 5 hours. Pump A starts filling up a balloon at 12:00 PM, and pump B is added alongside pump A at a later time. If the balloon is completely filled at 2:00 PM, how many minutes after 12:00 PM was Pump B added?

11. For some positive integer k, the product $81 \cdot k$ has 20 factors. Find the smallest possible value of k.

12. Two people wish to sit in a row of fifty chairs. How many ways can they sit in the chairs if they do not want to sit directly next to each other and they do not want to sit with exactly one empty chair between them?

13. Let $\triangle ABC$ be an equilateral triangle with side length 2 and M be the midpoint of BC. Let P be a point in the same plane such that $2PM = BC$. The minimum value of AP can be expressed as $\sqrt{a} - b$, where a and b are positive integers such that a is not divisible by any perfect square aside from 1. Find $a + b$.

14. What are the 2022nd to 2024th digits after the decimal point in the decimal expansion of $\frac{1}{27}$, expressed as a 3 digit number in that order (i.e the 2022nd digit is the hundreds digit, 2023rd digit is the tens digit, and 2024th digit is the ones digit)?

15. After combining like terms, how many terms are in the expansion of $(xyz + xy + yz + xz + x + y + z)^{20}$?

16. Let $ABCD$ be a trapezoid with $AB \parallel CD$ where $AB > CD$, $\angle B = 90^\circ$, and $BC = 12$. A line k is dropped from A, perpendicular to line CD, and another line ℓ is dropped from C, perpendicular to line AD. k and ℓ intersect at X. If $\triangle AXC$ is an equilateral triangle, the area of $ABCD$ can be written as $m\sqrt{n}$, where m and n are positive integers such that n is not divisible by any perfect square aside from 1. Find $m + n$.

17. If real numbers x and y satisfy $2x^2 + y^2 = 8x$, maximize the expression $x^2 + y^2 + 4x$.

18. Let $f(x)$ be a monic quadratic polynomial with nonzero real coefficients. Given that the minimum value of $f(x)$ is one of the roots of $f(x)$, and that $f(2022) = 1$, there are two possible values of $f(2023)$. Find the larger of these two values.

19. I am thinking of a positive integer. After realizing that it is four more than a multiple of 3, four less than a multiple of 4, four more than a multiple of 5, and four less than a multiple of 7, I forgot my number. What is the smallest possible value of my number?

20. How many ways can Aston, Bryan, Cindy, Daniel, and Evan occupy a row of 14 chairs such that none of them are sitting next to each other?

21. Let x be a positive real number. The minimum value of $\frac{1}{x^2} + \sqrt{x}$ can be expressed in the form $\frac{a}{b^{(c/d)}}$, where a, b, c, d are all positive integers, a and b are relatively prime, c and d are relatively prime, and b is not divisible by any perfect square. Find $a + b + c + d$.

22. For all $x > 0$, the function $f(x)$ is defined as $\lfloor x \rfloor \cdot (x + \{x\})$. There are 24 possible x such that $f(x)$ is an integer between 2000 and 2023, inclusive. If the sum of these 24 numbers equals N, then find $\lfloor N \rfloor$.

    Note: Recall that $\lfloor x \rfloor$ is the greatest integer less than or equal to x, called the *floor* function. Also, $\{x\}$ is defined as $x - \lfloor x \rfloor$, called the *fractional part* function.

23. Let $ABCD$ be a rectangle with $AD = 1$. Let P be a point on diagonal $\overline{AC}$, and let ω and ξ be the circumcircles of $\triangle APB$ and $\triangle CPD$, respectively. Line $\overleftrightarrow{AD}$ is extended, intersecting ω at X, and ξ at Y. If $AX = 5$ and $DY = 2$, find $[ABCD]^2$.

    Note: $[ABCD]$ denotes the area of the polygon $ABCD$.

24. Alice writes all of the three-digit numbers on a blackboard (it's a pretty big blackboard). Let X_a be the set of three-digit numbers containing a somewhere in its representation, where a is a string of digits. (For example, X_{12} would include 12, 121, 312, etc.) If Bob then picks a value of a at random so $0 \leq a \leq 999$, the expected number of elements in X_a can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

25. Let $f(x) = x^5 + 2x^4 - 2x^3 + 4x^2 + 5x + 6$ and $g(x) = x^4 - x^3 + x^2 - x + 1$. If a, b, c, d are the roots of $g(x)$, then find $f(a) + f(b) + f(c) + f(d)$.