Team Name:________________

Round 1

1. When Jay was 4 years old, his father was seven times older than him. How many years must pass such that Jay will be half as young as his father?
2. Aspen needs to use a 25-feet ladder to reach the top of a high wall. If the ladder needs to rest 7 feet from the base of the wall, how many inches tall is the wall?
3. Ben has four textbooks on his bookshelf: geometry, biology, trigonometry, and chemistry. If he has to study geometry before trigonometry, in how many orders can Ben study the four subjects?

1.________________ 2.________________ 3.________________

Team Name:________________

Round 2

1. In a right triangle $\triangle ABC$ with $\angle B = 90^\circ$, $AB = 6$ and $BC = 8$. The altitude from B is dropped to $\overline{AC}$ at point X. The length of $\overline{BX}$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime. Find $m + n$.
2. Find the smallest positive integer n that is a multiple of 15 and whose digits sum to 15.
3. Anna wants to build a snowman! She does this by randomly picking 15 snowballs in front of her, all of different sizes. She first picks a snowball for the lower body, another snowball for the torso, and a last snowball for the head. The probability the snowman's lower body is larger than its torso which is larger than the head can be expressed as $\frac{a}{b}$, where a and b are relatively prime. Find $a + b$.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 3

1. Larry is lifting weights. Given that the number of repetitions he can do of a weight (how many times he can lift the weight in a row) is inversely proportional to the weight and directly proportional to the amount of rest he gets, and Larry can lift 320 pounds 6 times with 2 minute rests, how many repetitions of 240 pound weights can Larry do if he has 3 minute rests?

2. Alex and Bob decide to go to an art gallery on the same day. The art gallery is open for walk-ins from $9:00$ AM to $12:00$ PM. Alex will enter to the art gallery from any time between $9:00$ AM to $11:30$ AM and stay there for 30 minutes. Bob will enter the art gallery any time from $10:00$ AM to $11:00$ AM and stays there for 30 minutes. The probability that both Alex and Bob will be in the gallery at the same time can be expressed as $\frac{p}{q}$, where p and q are relatively prime. Find $p+q$.

3. Consider a circle with center A and radius 8. Two tangents are constructed from points B and C on the circle to an outside point D. Two additional segments are constructed from A to points E and F on $\overline{BD}$ and $\overline{CD}$ respectively, such that $BE = \frac{2}{3}DE$ and $CF = \frac{2}{3}DF$. Lastly, the diameter with endpoints X and Y is constructed to be parallel to $\overline{BC}$. If $AE = 10$, find the area of $\triangle XDY$.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 4

1. If there exist real numbers x and y such that $4^x = 3125$ and $5^y = 4096$, find xy.

2. Andrew, Daniel, and Eric are going on a trip to Mexico costing \$120. Daniel will only pay in multiples of \$5, and Andrew refuses to pay more than Eric does. How many ways can the three friends pay?

3. Raymond is currently very sick. To combat this, he is taking a lot medicine: every day, he must eat 2 blue pills, 4 red pills, and 4 green pills. He wants to eat the pills two at a time, and the two pills he eats must not be the same color. How many days will it take him to eat the pills in every combination that he can possibly make?

1.________________ 2.________________ 3.________________

Team Name:________________

Round 5

1. Pat the Pirate is looking for gold in GoldLand. There are 6 islands in GoldLand, each numbered 1 through 6. On even-numbered islands, Pat can multiply his current pieces of gold by the number of the island. On odd-numbered islands, Pat can increase his current pieces of gold by the number of the island. Pat starts with no gold and he can choose the order of islands, visiting each island at most once in whatever order he chooses. If m is the minimum number of gold pieces Pat can end with, and M is the maximum, find $M - m$.

2. The value of the following sum

$$\binom{2024}{0} + \binom{2024}{4} + \binom{2024}{8} + ... + \binom{2024}{4k} + ... + \binom{2024}{2024}$$

can be expressed as $2^x + 2^y$, where x and y are positive integers. Find $x + y$.

Note: The notation $\binom{n}{k}$ expresses the number of ways to choose k objects from a group of n. For example, $\binom{5}{2} = 10$ because there are 10 ways to choose 2 objects from a group of 5. Additionally, $\binom{n}{0} = 1$ for any n.

3. Given real numbers a, b, c, and d, what is the minimum possible value of

$$\sqrt{(a-42)^2 + 1^2} + \sqrt{(b-a)^2 + 2^2} + \sqrt{(c-b)^2 + 3^2} + \sqrt{(d-c)^2 + 4^2} + \sqrt{(154-d)^2 + 5^2}?$$

1.________________ 2.________________ 3.________________

Team Name:________________

Round 6

1. The following sequence s contains 2024 terms, where the kth term is defined by $8k + 1$.

$$s = \{8 \times 1 + 1, 8 \times 2 + 1, 8 \times 3 + 1, \cdots, 8 \cdot 2024 + 1\}$$

How many perfect squares does s contain?

2. Let $\varphi = \frac{1}{2}(1 + \sqrt{5})$, the golden ratio.

A rhombus $ABCD$ is constructed such that $AC = 12$ and $BD = 12\varphi$. The circumcircle of $\triangle ABD$ is drawn. Lines $\overleftrightarrow{AC}$ and $\overleftrightarrow{CD}$ are extended to meet the circle at P and Q respectively. The area of $PQBC$ can be expressed as $a + b\sqrt{5}$, where a and b are positive integers. Find $a + b$.

3. Of the six roots of the equation $(x-4)(x-5)(x-7)(x-9)(x-11)(x-12) = -144$, two of them are not only integers, but also double roots. Find the value of the integer double root.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 7

1. Let $\overline{AB}$ be a diameter of the circle ω with radius r. Let point $C \neq A, B$ be on segment $\overline{AB}$ such that $AC = s$. The tangent line ℓ to ω at A contains a variable point P, and Q is defined as the point on ℓ such that $\overline{PC} \perp \overline{BQ}$. Lastly, the extension of line $\overleftrightarrow{AB}$ intersects the circumcircle of $\triangle BPQ$ at R.

   Let $F(r, s)$ be a function of r and s that denotes the length of $\overline{AP}$ for the minimum area of quadrilateral $BPRQ$. For how many ordered pairs of integers (r, s) with $1 \leq r \leq 20$ will $F(r, s)$ also be an integer?

2. Define the following infinite sequence s:

   $$s = \left\{ \frac{1}{5}, \frac{4}{25}, \frac{9}{125}, \dots, \frac{k^2}{5^k}, \dots \right\}.$$

   As more terms in s are summed, they approach a value denoted S. S can be expressed as a fraction $\frac{a}{b}$, where a and b are relatively prime. Find $a + b$.

3. Let $f(x) = (5x)!$ and $g(x) = 5^x(x!)$. Find the remainder of $f(31)/g(31)$ when it is divided by 25.

1.________________ 2.________________ 3.________________

Team Name:________________

Tiebreaker

The previous questions on the **Team Round** all have answers ranging from 0 to 100000. Let N be the number of problems on the **Team Round**, S be the sum of their answers, and P be the product of their answers.

Find the integer that is closest to $\log_S(P^N)$.

Your answer will be scored according to the following formula, where X is the correct answer and I is your input.

$$\max\left\{0, \left\lceil \min\left\{13 - \frac{|I - X|}{0.05|I|}, 13 - \frac{|I - X|}{0.05|I - 2X|}\right\}\right\rceil\right\}.$$

Answer:____________