Acton-Boxborough Math Competition Online Contest

Saturday, December 16 — Sunday, December 17, 2023

Contest Rules and Format

The 2023 December Contest consists of 15 problems worth 1 point each — each with an answer between 0 and 100,000. The contest window is

Saturday, December 16 to Sunday, December 17.

The following rules supersede any others regarding the Online Contests.

- To submit your responses, fill out a form on our website, `abmathcompetition.wordpress.com`. Include your email, name, school, grade (we require this information for prize purposes).
- Submissions will close at 10:00PM Eastern Standard Time, on the final day of the round.
- Computational aids such as WolframAlpha, calculators, abaci, and other related materials are prohibited. Graphing technology such as GeoGebra, Desmos, or a graphing calculator are also prohibited. Drawing aids such as rulers and protractors are permissible.
- You may only work individually on the problems—consultation with others is not permitted.
- You may take as much time as you wish during the contest window.
- Ties will be broken by the most difficult problem solved, determined by the number of correct answers to the question. If problem A is solved by a contestants, and problem B is solved by b contestants, with $a < b$, then problem A is said to be "more difficult" than B.

Awards and Prizes

- Top scoring contestants within each round will have their names posted on our website.
- Scores will be emailed to contestants within one week of the contest's end.
- Top scoring contestants across all three rounds will receive prizes.
- Physical prizes (such as AoPS gift certificates, merchandise, calculators, etc.) will be given to top scorers if they attend the ABMC onsite contest.

Thanks to our Sponsors!

Good luck!

Problems

1. Eric is playing Brawl Stars. If he starts playing at 11:10 AM, and plays for 2 hours total, then how many minutes past noon does he stop playing?

2. James is making a mosaic. He takes an equilateral triangle and connects the midpoints of its sides. He then takes the center triangle formed by the midsegments and connects the midpoints of its sides. In total, how many equilateral triangles are in James' mosaic?

3. What is the greatest amount of intersections that 3 circles and 3 lines can have, given that they all lie on the same plane?

4. In the faraway land of Arkesia, there are two types of currencies: Silvers and Gold. Each Silver is worth 7 dollars while each Gold is worth 17 dollars. In Daniel's wallet, the total dollar value of the Silvers is 1 more than that of the Golds. What is the smallest total dollar value of all of the Silvers and Golds in his wallet?

5. A bishop is placed on a random square of a 8-by-8 chessboard. On average, the bishop is able to move to s other squares on the chessboard. Find $4s$.

   Note: A *bishop* is a chess piece that can move diagonally in any direction, as far as it wants.

6. Andrew has a certain amount of coins. If he distributes them equally across his 9 friends, he will have 7 coins left. If he apportions his coins for each of his 15 classmates, he will have 13 coins to spare. If he splits the coins into 4 boxes for safekeeping, he will have 2 coins left over. What is the minimum number of coins Andrew could have?

7. A regular polygon P has three times as many sides as another regular polygon Q. The interior angle of P is 16° greater than the interior angle of Q. Compute how many *more* diagonals P has compared to Q.

8. In an certain airport, there are three ways to switch between the ground floor and second floor that are 30 meters apart: either stand on an escalator, run on an escalator, or climb the stairs.

   A family on vacation takes 65 seconds to climb up the stairs. A solo traveller late for their flight takes 25 seconds to run upwards on the escalator. The amount of time (in seconds) it takes for someone to switch floors by standing on the escalator can be expressed as $\frac{u}{v}$, where u and v are relatively prime. Find $u + v$.

   (Assume everyone has the same running speed, and the speed of running on an escalator is the sum of the speeds of riding the escalator and running on the stairs.)

9. Avanish, being the studious child he is, is taking practice tests to improve his score. Avanish has a 60% chance of passing a practice test. However, whenever Avanish passes a test, he becomes more confident and instead has a 70% chance of passing his next immediate test. If Avanish takes 3 practice tests in a row, the expected number of practice tests Avanish will pass can be expressed as $\frac{a}{b}$, where a and b are relatively prime. Find $a + b$.

10. Triangle $\triangle ABC$ has sides $AB = 51, BC = 119$, and $AC = 136$. Point C is reflected over line $\overline{AB}$ to create point C'. Next, point B is reflected over line $\overline{AC'}$ to create point B'. If $[B'C'C]$ can be expressed in the form of $a\sqrt{b}$, where b is not divisible by any perfect square besides 1, find $a + b$.

11. Define the following infinite sequence s:

    $$s = \left\{ \frac{1}{1}, \frac{1}{1+3}, \frac{1}{1+3+6}, \cdots, \frac{1}{1+3+6+\cdots+t_k}, \cdots \right\},$$

    where t_k denotes the kth triangular number. The sum of the first 2024 terms of s, denoted S, can be expressed as

    $$S = 3\left(\frac{1}{2} + \frac{1}{a} - \frac{1}{b}\right),$$

where a and b are positive integers. Find the minimal possible value of $a + b$.

Note: The kth triangular number is formed by summing the first k natural numbers. For example, the sixth triangular number is $1 + 2 + 3 + 4 + 5 + 6 = 21$.

12. Omar writes the numbers from 1 to 1296 on a whiteboard and then converts each of them into base 6. Find the sum of all of the digits written on the whiteboard (in base 10), including both the base 10 and base 6 numbers.

13. A *mountain number* is a number in a list that is greater than the number to its left and right. Let N be the amount of lists created from the integers $1 - 100$ such that each list only has one mountain number. N can be expressed as

$$N = 2(a \cdot 2^b + 1),$$

where a and b are integers. Find $a + b$.

(The numbers at the beginning or end of a list are not considered mountain numbers.)

14. A circle ω with center O has a radius of 25. Chords $\overline{AB}$ and $\overline{CD}$ are drawn in ω, intersecting at X such that $\angle BXC = 60^\circ$ and $AX > BX$. Given that the shortest distance of O with $\overline{AB}$ and $\overline{CD}$ is 7 and 15 respectively, the length of BX can be expressed as $x - \frac{y}{\sqrt{z}}$, where x, y, and z are positive integers such that z is not divisible by any perfect square. Find $x + y + z$.

15. How many ways are there to split the first 10 natural numbers into n sets (with $n \geq 1$) such that all the numbers are used and each set has the same average?