

Team Name:\_\_\_\_\_

### Round 1

1. If the sum of two non-zero integers is 28, then find the largest possible ratio of these integers.
2. If Tom rolls a eight-sided die where the numbers 1 – 8 are all on a side, let  $\frac{m}{n}$  be the probability that the number is a factor of 16 where  $m, n$  are relatively prime positive integers. Find  $m + n$ .
3. The average score of 35 second graders on an IQ test was 180 while the average score of 70 adults was 90. What was the total average IQ score of the adults and kids combined?

1.\_\_\_\_\_ 2.\_\_\_\_\_ 3.\_\_\_\_\_

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### Round 2

1. So far this year, Bob has gotten a 95 and a 98 in Term 1 and Term 2. How many different pairs of Term 3 and Term 4 grades can Bob get such that he finishes with an average of 97 for the whole year? Bob can only get integer grades between 0 and 100, inclusive.
2. If a complement of an angle M is one-third the measure of its supplement, then what would be the measure (in degrees) of the third angle of an isosceles triangle in which two of its angles were equal to the measure of angle M?
3. The distinct symbols ♡, ◇, ♣ and ♠ each correlate to one of +, −, ×, ÷, not necessarily in that given order. Given that

$$((((72 \diamond 36) \spadesuit 0) \diamond 32) \clubsuit 3) \heartsuit 2 = 6,$$

what is the value of

$$((((((64 \spadesuit 8) \heartsuit 6) \clubsuit 5) \heartsuit 1) \clubsuit 7) \diamond 1)?$$

1.\_\_\_\_\_ 2.\_\_\_\_\_ 3.\_\_\_\_\_

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**Round 3**

1. How many ways can 5 bunnies be chosen from 7 male bunnies and 9 female bunnies if a majority of female bunnies is required? All bunnies are distinct from each other.
2. If the product of the LCM and GCD of two positive integers is 2021, what is the product of the two positive integers?
3. The month of April in ABMC-land is 50 days long. In this month, on 44% of the days it rained, and on 28% of the days it was sunny. On half of the days it was sunny, it rained as well. The rest of the days were cloudy. How many days were cloudy in April in ABMC-land?

1.\_\_\_\_\_ 2.\_\_\_\_\_ 3.\_\_\_\_\_

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**Round 4**

1. In how many ways can 4 distinct dice be rolled such that a sum of 10 is produced?
2. If  $p, q, r$  are positive integers such that  $p^3\sqrt{qr^2} = 50$ , find the sum of all possible values of  $pqr$ .
3. Given that numbers  $a, b, c$  satisfy  $a + b + c = 0$ ,  $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = 10$ , and  $ab + bc + ac \neq 0$ , compute the value of  $\frac{-a^2 - b^2 - c^2}{ab + bc + ac}$ .

1.\_\_\_\_\_ 2.\_\_\_\_\_ 3.\_\_\_\_\_

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**Round 5**

1. A circle with a radius of 1 is inscribed in a regular hexagon. This hexagon is inscribed in a larger circle. If the area that is outside the hexagon but inside the larger circle can be expressed as  $\frac{a\pi}{b} - c\sqrt{d}$ , where  $a, b, c, d$  are positive integers,  $a, b$  are relatively prime, and no prime perfect square divides into  $d$ . find the value of  $a + b + c + d$ .
2. At a dinner party, 10 people are to be seated at a round table. If person A cannot be seated next to person B and person C must be next to person D, how many ways can the 10 people be seated? Consider rotations of a configuration identical.
3. Let  $N$  be the sum of all the positive integers that are less than 2022 and relatively prime to 1011. Find  $\frac{N}{2022}$ .

1. \_\_\_\_\_ 2. \_\_\_\_\_ 3. \_\_\_\_\_

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**Round 6**

1. The line  $y = m(x - 6)$  passes through the point  $A (6, 0)$ , and the line  $y = 8 - \frac{x}{m}$  pass through point  $B (0, 8)$ . The two lines intersect at point  $C$ . What is the largest possible area of triangle  $ABC$ ?
2. Let  $N$  be the number of ways there are to arrange the letters of the word *MATHEMATICAL* such that no two  $A$ s can be adjacent. Find the last 3 digits of  $\frac{N}{100}$ .
3. Find the number of ordered triples of integers  $(a, b, c)$  such that  $|a|, |b|, |c| \leq 100$  and  $3abc = a^3 + b^3 + c^3$ .

1. \_\_\_\_\_ 2. \_\_\_\_\_ 3. \_\_\_\_\_

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### Round 7

1. In a given plane, let  $A, B$  be points such that  $AB = 6$ . Let  $S$  be the set of points such that for any point  $C$  in  $S$ , the circumradius of  $\triangle ABC$  is at most 6. Find  $a + b + c$  if the area of  $S$  can be expressed as  $a\pi + b\sqrt{c}$  where  $a, b, c$  are positive integers, and  $c$  is not divisible by the square of any prime.
2. Compute  $\sum_{1 \leq a < b < c \leq 7} abc$ .
3. Three identical circles are centered at points  $A, B$ , and  $C$  respectively and are drawn inside a unit circle. The circles are internally tangent to the unit circle and externally tangent to each other. A circle centered at point  $D$  is externally tangent to circles  $A, B$ , and  $C$ . If a circle centered at point  $E$  is externally tangent to circles  $A, B$ , and  $D$ , what is the radius of circle  $E$ ? The radius of circle  $E$  can be expressed as  $\frac{a\sqrt{b}-c}{d}$  where  $a, b, c$ , and  $d$  are all positive integers,  $\gcd(a, c, d) = 1$ , and  $b$  is not divisible by the square of any prime. What is the sum of  $a + b + c + d$ ?

1. \_\_\_\_\_ 2. \_\_\_\_\_ 3. \_\_\_\_\_

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### Round 8

Let  $A$  be the number of unused Algebra problems in our problem bank. Let  $B$  be the number of times the letter 'b' appears in our problem bank. Let  $M$  be the median speed round score. Finally, let  $C$  be the number of correct answers to Speed Round 1. Estimate

$$A \cdot B + M \cdot C.$$

Your answer will be scored according to the following formula, where  $X$  is the correct answer and  $I$  is your input.

$$\max \left\{ 0, \left\lceil \min \left\{ 13 - \frac{|I - X|}{0.05|I|}, 13 - \frac{|I - X|}{0.05|I - 2X|} \right\} \right\rceil \right\}.$$

Answer:\_\_\_\_\_