Team Name:________________

Round 1

1. There are 99 dogs sitting in a long line. Starting with the third dog in the line, if every third dog barks three times, and all the other dogs each bark once, how many barks are there in total?
2. Indigo notices that when she uses her lucky pencil, her test scores are always $66\frac{2}{3}\%$ higher than when she uses normal pencils. What percent lower is her test score when using a normal pencil than her test score when using her lucky pencil?
3. Bill has a farm with deer, sheep, and apple trees. He mostly enjoys looking after his apple trees, but somehow, the deer and sheep always want to eat the trees' leaves, so Bill decides to build a fence around his trees. The 60 trees are arranged in a 5×12 rectangular array with 5 feet between each pair of adjacent trees. If the rectangular fence is constructed 6 feet away from the array of trees, what is the area the fence encompasses in feet squared? (Ignore the width of the trees.)

1.________________ 2.________________ 3.________________

Team Name:________________

Round 2

1. If x + 3y = 2, then what is the value of the expression: $9^x * 729^y$?
2. Lazy Sheep loves sleeping in, but unfortunately, he has school two days a week. If Lazy Sheep wakes up each day before school's starting time with probability 1/8 independent of previous days, then the probability that Lazy Sheep wakes up late on at least one school day over a given week is $\frac{p}{q}$ for relatively prime positive integers p, q. Find $p + q$.
3. An integer n leaves remainder 1 when divided by 4. Find the sum of the possible remainders n leaves when divided by 20.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 3

1. Jake has a circular knob with three settings that can freely rotate. Each minute, he rotates the knob 120° clockwise or counterclockwise at random. The probability that the knob is back in its original state after 4 minutes is $\frac{p}{q}$ for relatively prime positive integers p, q. Find $p + q$.
2. Given that 3 not necessarily distinct primes p, q, r satisfy $p + 6q + 2r = 60$, find the sum of all possible values of $p + q + r$.
3. Dexter's favorite number is the positive integer x. If $15x$ has an even number of proper divisors, what is the smallest possible value of x? (Note: A proper divisor of a positive integer is a divisor other than itself.)

1.________________ 2.________________ 3.________________

Team Name:________________

Round 4

1. Three circles of radius 1 are each tangent to the other two circles. A fourth circle is externally tangent to all three circles. The radius of the fourth circle can be expressed as $\frac{a\sqrt{b} - c}{d}$ for positive integers a, b, c, d where b is not divisible by the square of any prime and a and d are relatively prime. Find $a + b + c + d$.
2. Evaluate $\frac{\sqrt{15}}{3} \cdot \frac{\sqrt{35}}{5} \cdot \frac{\sqrt{63}}{7} \cdot \dots \cdot \frac{\sqrt{5475}}{73}$.
3. For any positive integer n, let $f(n)$ denote the number of digits in its base 10 representation, and let $g(n)$ denote the number of digits in its base 4 representation. For how many n is $g(n)$ an integer multiple of $f(n)$?

1.________________ 2.________________ 3.________________

Team Name:________________

Round 5

1. Julia baked a pie for herself to celebrate pi day this year. If Julia bakes anyone pie on pi day, the following year on pi day she bakes a pie for herself with 1/3 probability, she bakes her friend a pie with 1/6 probability, and she doesn't bake anyone a pie with 1/2 probability. However, if Julia doesn't make pie on pi day, the following year on pi day she bakes a pie for herself with 1/2 probability, she bakes her friend a pie with 1/3 probability, and she doesn't bake anyone a pie with 1/6 probability. The probability that Julia bakes at least 2 pies on pi day in the next 5 years can be expressed as $\frac{p}{q}$, for relatively prime positive integers p and q. Compute $p+q$.
2. Steven is flipping a coin but doesn't want to appear too lucky. If he flips the coin 8 times, the probability he only gets sequences of consecutive heads or consecutive tails that are of length 4 or less can be expressed as $\frac{p}{q}$, for relatively prime positive integers p and q. Compute $p+q$.
3. Let $ABCD$ be a square with side length 3. Further, let E be a point on side AD, such that $AE = 2$ and $DE = 1$, and let F be the point on side AB such that triangle CEF is right with hypotenuse CF. The value CF^2 can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m+n$.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 6

1. Let P be a point outside circle ω with center O. Let A, B be points on circle ω such that PB is a tangent to ω and $PA = AB$. Let M be the midpoint of AB. Given $OM = 1, PB = 3$, the value of AB^2 can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n. Find $m+n$.
2. Let $a_0, a_1, a_2 ...$ with each term defined as $a_n = 3a_{n-1} + 5a_{n-2}$ and $a_0 = 0, a_1 = 1$. Find the remainder when a_{2020} is divided by 360.
3. James and Charles each randomly pick two points on distinct sides of a square, and they each connect their chosen pair of points with a line segment. The probability that the two line segments intersect can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n. Find $m+n$.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 7

1. For some positive integers x, y let $g = \gcd(x, y)$ and $\ell = \operatorname{lcm}(2x, y)$. Given that the equation $xy + 3g + 7\ell = 168$ holds, find the largest possible value of $2x + y$.

2. Marco writes the polynomials $f(x) = nx^4 + 2x^3 + 3x^2 + 4x + 5$ and $g(x) = a(x-1)^4 + b(x-1)^3 + 6(x-1)^2 + d(x-1) + e$, where n, a, b, d, e are real numbers. He notices that $g(i) = f(i) - |i|$ for each integer i satisfying $-5 \le i \le -1$. Then n^2 can be expressed as $\frac{p}{q}$ for relatively prime positive integers p, q. Find $p + q$.

3. Equilateral $\triangle ABC$ is inscribed in a circle with center O. Points D and E are chosen on minor arcs AB and BC, respectively. Segment $\overline{CD}$ intersects $\overline{AB}$ and $\overline{AE}$ at Y and X, respectively. Given that $\triangle DXE$ and $\triangle AXC$ have equal area, $\triangle AXY$ has area 1, and $\triangle ABC$ has area 52, find the area of $\triangle BXC$.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 8

Let A be the number of total webpage visits our website received last month. Let B be the number photos in our photo collection from ABMC onsite 2017. Let M be the mean speed round score. Further, let C be the number of times the letter c appears in our problem bank. Estimate

$$A \cdot B + M \cdot C.$$

Your answer will be scored according to the following formula, where X is the correct answer and I is your input.

$$\max\left\{0, \left\lceil \min\left\{13 - \frac{|I-X|}{0.05|I|}, 13 - \frac{|I-X|}{0.05|I-2X|}\right\}\right\rceil\right\}.$$

Answer:____________