Name: ______________________ **Score:** ________/________/50 **Grade:** ______

The speed round is 30 minutes long. Questions are weighted by difficulty.

1. ________	6. ________	11. ________	16. ________	21. ________
2. ________	7. ________	12. ________	17. ________	22. ________
3. ________	8. ________	13. ________	18. ________	23. ________
4. ________	9. ________	14. ________	19. ________	24. ________
5. ________	10. ________	15. ________	20. ________	25. ________

Special Thanks to:

1. You and nine friends spend 4000 dollars on tickets to attend the new Harry Styles concert. Unfortunately, six friends cancel last minute due to the flu. You and your remaining friends still attend the concert and split the original cost of 4000 dollars equally. What percent of the total cost does each remaining individual have to pay?

2. Find the number distinct 4 digit numbers that can be formed by arranging the digits of 2021.

3. On a plane, Darnay draws a triangle and a rectangle such that each side of the triangle intersects each side of the rectangle at no more than one point. What is the largest possible number of points of intersection of the two shapes?

4. Joy is thinking of a two-digit number. Her hint is that her number is the sum of two 2-digit perfect squares x_1 and x_2 such that exactly one of $x_i - 1$ and $x_i + 1$ is prime for each $i = 1, 2$. What is Joy's number?

5. At the North Pole, ice tends to grow in parallelogram structures of area 60. On the other hand, at the South Pole, ice grows in right triangular structures, in which each triangular and parallelogram structure have the same area. If every ice triangle ABC has legs $\overline{AB}$ and $\overline{AC}$ that are integer lengths, how many distinct possible lengths are there for the hypotenuse $\overline{BC}$?

6. Carlsen has some squares and equilateral triangles, all of side length 1. When he adds up the interior angles of all shapes, he gets 1800°. When he adds up the perimeters of all shapes, he gets 24. How many squares does he have?

7. Vijay wants to hide his gold bars by melting and mixing them into a water bottle. He adds 100 grams of liquid gold to 100 grams of water. His liquefied gold bars have a density of 20 g/ml and water has a density of 1 g/ml. Given that the density of the mixture in g/mL can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n, compute the sum $m + n$. (Note: density is mass divided by volume, gram (g) is unit of mass and ml is unit of volume. Further, assume the volume of the mixture is the sum of the volumes of the components.)

8. Julius Caesar has epilepsy. Specifically, if he sees 3 or more flashes of light within a 0.1 second time frame, he will have a seizure. His enemy Brutus has imprisoned him in a room with 4 screens, which flash exactly every 4, 5, 6, and 7 seconds, respectively. The screens all flash at once, and 105 seconds later, Caesar opens his eyes. How many seconds after he opened his eyes will Caesar first get a seizure?

9. Angela has a large collection of glass statues. One day, she was bored and decided to use some of her statues to create an entirely new one. She melted a sphere with radius 12 and a cone with height of 18 and base radius of 2. If Angela wishes to create a new cone with a base radius 2, what would the the height of the newly created cone be?

10. Find the smallest positive integer N satisfying these properties:
    (a) No perfect square besides 1 divides N.
    (b) N has exactly 16 positive integer factors.

11. The probability of a basketball player making a free throw is $\dfrac{1}{5}$. The probability that she gets exactly 2 out of 4 free throws in her next game can be expressed as $\dfrac{m}{n}$ for relatively prime positive integers m and n. Find $m + n$.

12. A new donut shop has 1000 boxes of donuts and 1000 customers arriving. The boxes are numbered 1 to 1000. Initially, all boxes are lined up by increasing numbering and closed. On the first day of opening, the first customer enters the shop and opens all the boxes for taste testing. On the second day of opening, the second customer enters and closes every box with an even number. The third customer then "reverses" (if closed, they open it and if open, they close it) every box numbered with a multiple of three, and so on, until all 1000 customers get kicked out for having entered the shop and reversing their set of boxes. What is the number on the sixth box that is left open?

13. For an assignment in his math class, Michael must stare at an analog clock for a period of 7 hours. He must record the times at which the minute hand and hour hand form an angle of exactly $90°$, and he will receive 1 point for every time he records correctly. What is the maximum number of points Michael can earn on his assignment?

14. The graphs of $y = x^3 + 5x^2 + 4x - 3$ and $y = -\frac{1}{5}x + 1$ intersect at three points in the Cartesian plane. Find the sum of the y-coordinates of these three points.

15. In the quarterfinals of a single elimination countdown competition, the 8 competitors are all of equal skill. When any 2 of them compete, there is exactly a 50% chance of either one winning. If the initial bracket is randomized, the probability that two of the competitors, Daniel and Anish, face off in one of the rounds can be expressed as $\frac{p}{q}$ for relatively prime positive integers p, q. Find $p + q$.

16. How many positive integers less than or equal to 1000 are not divisible by any of the numbers $2, 3, 5$ and 11?

17. A strictly increasing geometric sequence of positive integers $a_1, a_2, a_3 \cdots$ satisfies the following properties:

    (a) Each term leaves a common remainder when divided by 7

    (b) The first term is an integer from 1 to 6

    (c) The common ratio is an perfect square

    Let N be the smallest possible value of $\frac{a_{2021}}{a_1}$. Find the remainder when N is divided by 100.

18. Suppose $p(x) = x^3 - 11x^2 + 36x - 36$ has roots r, s, t. Find $\frac{r^2 + s^2}{t} + \frac{s^2 + t^2}{r} + \frac{t^2 + r^2}{s}$.

19. Let $a, b \leq 2021$ be positive integers. Given that ab^2 and a^2b are both perfect squares, let $G = \gcd(a, b)$. Find the sum of all possible values of G.

20. Jessica rolls six fair standard six-sided dice at the same time. Given that she rolled at least four 2's and exactly one 3, the probability that all six dice display prime numbers can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n. What is $m + n$?

21. Let a, b, c be numbers such $a + b + c$ is real and the following equations hold:

$$a^3 + b^3 + c^3 = 25,$$
$$\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ac} = 1,$$
$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{25}{9}.$$

    The value of $a + b + c$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n. Find $m + n$.

22. Let ω be a circle and P be a point outside ω. Let line ℓ pass through P and intersect ω at points A, B and with $PA < PB$ and let m be another line passing through P intersecting ω at points C, D with $PC < PD$. Let X be the intersection of AD and BC. Given that $\frac{PC}{CD} = \frac{2}{3}$, $\frac{PC}{PA} = \frac{4}{5}$, and $\frac{[ABC]}{[ACD]} = \frac{7}{9}$, the value of $\frac{[BXD]}{[BXA]}$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n. Find $m + n$.

23. Define the operation $a \circ b = \frac{a^2 + 2ab + a - 12}{b}$. Given that $1 \circ (2 \circ (3 \circ (\cdots 2019 \circ (2020 \circ 2021)))...)$ can be expressed as $-\frac{a}{b}$ for some relatively prime positive integers a, b, compute $a + b$.

24. Find the largest integer $n \leq 2021$ for which $5^{n-3} \mid (n!)^4$.

25. On the Cartesian plane, a line ℓ intersects a parabola with a vertical axis of symmetry at $(0, 5)$ and $(4, 4)$. The focus F of the parabola lies below ℓ, and the distance from F to ℓ is $\frac{16}{\sqrt{17}}$. Let the vertex of the parabola be (x, y). The sum of all possible values of y can be expressed as $\frac{p}{q}$ for relatively prime positive integers p, q. Find $p + q$.