Name: ______________________ **Score:** ______/______/50 **Grade:** ______

The accuracy round is 40 minutes long. Questions are weighted by difficulty.

1. ____________

2. ____________

3. ____________

4. ____________

5. ____________

6. ____________

7. ____________

8. ____________

9. ____________

10. ____________

11. ____________

Special Thanks to:

1. There is a string of numbers $1234567891023\ldots910134\ldots91012\ldots$ that concatenates the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$, then $2, 3, 4, 5, 6, 7, 8, 9, 10, 1$, then $3, 4, 5, 6, 7, 8, 9, 10, 1, 2$, and so on. After $10, 1, 2, 3, 4, 5, 6, 7, 8, 9$, the string will be concatenated with $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ again. What is the 2021st digit?

2. Bob really likes eating rice. Bob starts eating at the rate of 1 bowl of rice per minute. Every minute, the number of bowls of rice Bob eats per minute increases by 1. Given there are 78 bowls of rice, find number of minutes Bob needs to finish all the rice.

3. Suppose John has 4 fair coins, one red, one blue, one yellow, one green. If John flips all 4 coins at once, the probability he will land exactly 3 heads and land heads on both the blue and red coins can be expressed as $\frac{a}{b}$ for relatively prime positive integers a, b. Find $a + b$.

4. Three of the sides of an isosceles trapezoid have lengths $1, 10, 20$. Find the sum of all possible values of the fourth side.

5. An number *two-three-delightful* if and only if it can be expressed as the product of 2 consecutive integers larger than 1 and as the product of 3 consecutive integers larger than 1. What is the smallest two-three-delightful number?

6. There are 3 students total in Justin's online chemistry class. On a 100 point test, Justin's two classmates scored 4 and 7 points. The teacher notices that the class median score is equal to $\gcd(x, 42)$, where the positive integer x is Justin's score. Find the sum of all possible values of Justin's score.

7. Eddie's gym class of 10 students decides to play ping pong. However, there are only 4 tables and only 2 people can play at a table. If 8 students are randomly selected to play and randomly assigned a partner to play against at a table, the probability that Eddie plays against Allen is $\frac{a}{b}$ for relatively prime positive integers a, b. Find $a + b$.

8. Let S be the set of integers k consisting of nonzero digits, such that $300 < k < 400$ and $k - 300$ is not divisible by 11. For each k in S, let $A(k)$ denote the set of integers in S not equal to k that can be formed by permuting the digits of k. Find the number of integers k in S such that k is relatively prime to all elements of $A(k)$.

9. In $\triangle ABC, AB = 6$ and $BC = 5$. Point D is on side AC such that BD bisects angle $\angle ABC$. Let E be the foot of the altitude from D to AB. Given $BE = 4$, find AC^2.

10. For each integer $1 \le n \le 10$, Abe writes the number $2^n + 1$ on a blackboard. Each minute, he takes two numbers a and b, erases them, and writes $\frac{ab-1}{a+b-2}$ instead. After 9 minutes, there is one number C left on the board. The minimum possible value of C can be expressed as $\frac{p}{q}$ for relatively prime positive integers p, q. Find $p + q$.

11. **Estimation (Tiebreaker):** Let A and B be the proportions of contestants that correctly answered Questions 9 and 10 of this round, respectively. Estimate $\left\lfloor \frac{1}{(AB)^2} \right\rfloor$.