Team Name:________________

Round 1

1. A person asks for help every 3 seconds. Over a time period of 5 minutes, how many times will they ask for help?
2. In a big bag, there are 14 red marbles, 15 blue marbles, and 16 white marbles. If Anuj takes a marble out of the bag each time without replacement, how many marbles does Anuj need to remove to be sure that he will have at least 3 red marbles?
3. If Josh has 5 distinct candies, how many ways can he pick 3 of them to eat?

1.________________ 2.________________ 3.________________

Team Name:________________

Round 2

1. Annie has a circular pizza. She makes 4 straight cuts. What is the minimum number of slices of pizza that she can make?
2. What is the sum of the first 4 prime numbers that can be written as the sum of two perfect squares?
3. Consider a regular octagon ABCDEFGH inscribed in a circle of area 64π. If the length of arc ABC is $n\pi$, what is n?

1.________________ 2.________________ 3.________________

Team Name:________________

Round 3

1. Let ABCDEF be an equiangular hexagon with consecutive sides of length 6, 5, 3, 8, and 3. Find the length of the sixth side.
2. Jack writes all of the integers from 1 to n on a blackboard except the even primes. He selects one of the numbers and erases all of its digits except the leftmost one. He adds up the new list of numbers and finds that the sum is 2020. What was the number he chose?
3. Our original competition date was scheduled for April 11, 2020 which is a Saturday. The numbers 4116 and 2020 have the same remainder when divided by x. If x is a prime number, find the sum of all possible x.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 4

1. The polynomials $5p^2 + 13pq + cq^2$ and $5p^2 + 13pq - cq^2$ where c is a positive integer can both be factored into linear binomials with integer coefficients. Find c.
2. In a Cartesian coordinate plane, how many ways are there to get from $(0, 0)$ to $(2, 3)$ in 7 moves, if each move consists of a moving one unit either up, down, left, or right?
3. Bob the Builder is building houses. On Monday he finds an empty field. Each day starting on Monday, he finishes building a house at noon. On the nth day, there is a $\frac{n}{8}$ chance that a storm will appear at 3:14 PM and destroy all the houses on the field. At any given moment, Bob feels sad if and only if there is exactly 1 house left on the field that is not destroyed. The probability that he will not be sad on Friday at 6PM can be expressed as $\dfrac{p}{q}$ in simplest form. Find $p + q$.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 5

1. Quadrilateral ABCD is such that $\angle ABC = \angle ADC = 90^\circ$, $\angle BAD = 150^\circ$, $AD = 3$, and $AB = \sqrt{3}$. The area of ABCD can be expressed as $p\sqrt{q}$ for positive integers p, q where q is not divisible by the square of any prime. Find $p + q$.

2. Neetin wants to gamble, so his friend Akshay describes a game to him. The game will consist of three dice: a 100-sided one with the numbers 1 to 100, a tetrahedral one with the numbers 1 to 4, and a normal 6-sided die. If Neetin rolls numbers with a product that is divisible by 21, he wins. Otherwise, he pays Akshay 100 dollars. The number of dollars that Akshay must pay Neetin for a win in order to make this game fair is $\frac{a}{b}$ for relatively prime positive integers a, b. Find $a + b$. (Fair means the expected net gain is \$0.)

3. What is the sum of the fourth powers of the roots of the polynomial $P(x) = x^2 + 2x + 3$?

1.________________ 2.________________ 3.________________

Team Name:________________

Round 6

1. Consider the set $\mathbb{S} = \{1, 2, 3, 4 \ldots 25\}$. How many ordered n-tuples $S_1 = (a_1, a_2, a_3 \ldots a_n)$ of pairwise distinct a_i exist such that $a_i \in \mathbb{S}$ and $i^2 \mid a_i$ for all $1 \leq i \leq n$?

2. How many ways are there to place 2 identical rooks and 1 queen on a 4 x 4 chessboard such that no piece attacks another piece? (A queen can move diagonally, vertically or horizontally and a rook can move vertically or horizontally)

3. Let L be an ordered list $l_1, l_2, \ldots, l_{36}$ of consecutive positive integers who all have the sum of their digits not divisible by 11. It is given that l_1 is the least element of L. Find the least possible value of l_1.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 7

1. Spencer, Candice, and Heather love to play cards, but they especially love the highest cards in the deck - the face cards (jacks, queens, and kings). They also each have a unique favorite suit: Spencer's favorite suit is spades, Candice's favorite suit is clubs, and Heather's favorite suit is hearts. A dealer pulls out the 9 face cards from every suit except the diamonds and wants to deal them out to the 3 friends. How many ways can he do this so that none of the 3 friends will see a single card that is part of their favorite suit?

2. Suppose a sequence of integers satisfies the recurrence $a_{n+3} = 7a_{n+2} - 14a_{n+1} + 8a_n$. If $a_0 = 4$, $a_1 = 9$, and $a_2 = 25$, find a_{16}. Your answer will be in the form $2^a + 2^b + c$, where $2^a < a_{16} < 2^{a+1}$ and b is as large as possible. Find $a + b + c$.

3. Parallel lines l_1 and l_2 are 1 unit apart. Unit square $WXYZ$ lies in the same plane with vertex W on l_1. Line l_2 intersects segments YX and YZ at points U and O, respectively. Given $UO = \frac{9}{10}$, the inradius of $\triangle YOU$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n. Find $m + n$.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 8

Let A be the number of contestants who participated in at least one of the three rounds of the 2020 ABMC April contest. Let B be the number of times the letter b appears in the Accuracy Round. Let M be the number of people who submitted both the speed and accuracy rounds before 2:00 PM EST. Further, let C be the number of times the letter c appears in the Speed Round. Estimate

$$A \cdot B + M \cdot C.$$

Your answer will be scored according to the following formula, where X is the correct answer and I is your input.

$$\max\left\{0, \left\lceil \min\left\{13 - \frac{|I - X|}{0.05|I|}, 13 - \frac{|I - X|}{0.05|I - 2X|}\right\}\right\rceil\right\}.$$

Answer:____________