Acton-Boxborough Math Competition 2020 Solutions

ABMC Team

April 26, 2020

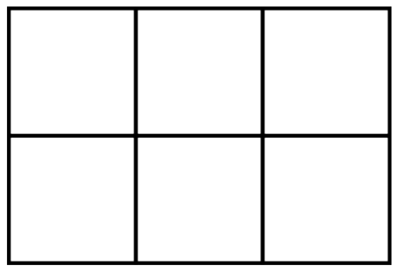
Speed Round

1. **Problem:** Today is Saturday, April 25, 2020. What is the value of $6 + 4 + 25 + 2020$?

   Solution: The value of $6 + 4 + 25 + 2020$ is $\boxed{2055}$

   Proposed by Aaron Zhang

2. **Problem:** The figure below consists of a 2 by 3 grid of squares. How many squares of any size are in the grid?

   Solution: There are 6 one-by-one squares and 2 two-by-two squares, so our answer is $\boxed{8}$. We are also accepting $\boxed{10}$ to include the 2 squares tilted by $45°$.

   Proposed by Aaron Zhang

3. **Problem:** James is playing a game. He first rolls a six-sided dice which contains a different number on each side, then randomly picks one of twelve different colors, and finally flips a quarter. How many different possible combinations of a number, a color and a flip are there in this game?

   Solution: Each combination consists of 1 of 6 numbers, 1 of 12 colors, and 1 of 2 sides of the quarter, so there are a total of $6 \times 12 \times 2 = \boxed{144}$ possible combinations.

   Proposed by Aaron Zhang

4. **Problem:** What is the sum of the number of diagonals and sides in a regular hexagon?

   Solution: The number of diagonals in any convex polygon with n sides is $\frac{(n)(n-3)}{2}$, so there are $\frac{(6)(3)}{2} = 9$ diagonals in a regular hexagon. Adding the 6 sides of the hexagon, we get a total of $\boxed{15}$.

   Proposed by Aaron Zhang

5. **Problem:** Mickey Mouse and Minnie Mouse are best friends but they often fight. Each of their fights take up exactly one hour, and they always fight on prime days. For example, they fight on January 2nd, 3rd, but not the 4th. Knowing this, how many total times do Mickey and Minnie fight in the months of April, May and June?

   Solution: Note that there are 10 prime numbers less than $31 : 2, 3, 5, 7, 11, 13, 17, 19, 23, 29$. Then since April and June have 30 days, Mickey and Minnie fight 20 times during those two months. May has 31 days, so Mickey and Minnie fight 11 times since 31 is also prime. The total number of times they fight, then, is $20 + 11 = \boxed{31}$.

   Proposed by Annie Wang

6. **Problem:** Apple always loved eating watermelons. Normal watermelons have around 13 black seeds and 25 brown seeds, whereas strange watermelons had 45 black seeds and 2 brown seeds. If Apple bought 14 normal watermelons and 7 strange watermelons, then let a be the total number of black seeds and b be the total number of brown seeds. What is $a - b$?

Solution: There are $13 \cdot 14 + 45 \cdot 7 = 182 + 315 = 497$ black seeds and $25 \cdot 14 + 2 \cdot 7 = 350 + 14 = 364$ brown seeds. Thus the answer is $497 - 364 = \boxed{133}$.

Proposed by Poonam Sahoo

7. **Problem:** Jerry and Justin both roll a die once. The probability that Jerry's roll is greater than Justin's can be expressed as a fraction in the form $\frac{m}{n}$ in simplified terms. What is $m + n$?

   Solution: First, we can calculate the probability that Jerry and Justin roll the same number. Regardless of what Jerry rolls, Justin will have a $\frac{1}{6}$ chance of rolling that same number, so the probability that they roll the same number is $\frac{1}{6}$. Next, notice that the probability that Jerry rolls a greater number is the same as the probability that Justin rolls a greater number than Jerry. Since these are the only three possible outcomes, the probability of Jerry's roll being greater than Justin's roll must be $\frac{1}{2}(1 - \frac{1}{6}) = \frac{5}{12}$ so the answer is $\boxed{17}$.

   Proposed by Aaron Zhang

8. **Problem:** Taylor wants to color the sides of an octagon. What is the minimum number of colors Taylor will need so that no adjacent sides of the octagon will be filled in with the same color?

   Solution: Suppose we start with the color red. Then, starting from the top and going clockwise, we can color every other side of the octagon (1st, 3rd, 5th, and 7th) with the color red. We cannot color any even side red because it is adjacent to two odd sides. Then we can choose another color to color the 2nd, 4th, 6th, and 8th sides. Thus the minimum number of colors needed is simply $\boxed{2}$.

   Proposed by Justin Shan

9. **Problem:** The point $\frac{2}{3}$ of the way from $(-6, 8)$ to $(-3, 5)$ can be expressed as an ordered pair (a, b). What is $|a - b|$?

   Solution: Note that $(-3, 5)$ is 3 units right and 3 units below $(-6, 8)$. Thus the point $\frac{2}{3}$ of the way from $(-6, 8)$ to $(-3, 5)$ would be 2 units right and 2 units below $(-6, 8)$, which gives (a, b) is $(-4, 6)$. Then $|a - b| = \boxed{10}$.

   Proposed by Poonam Sahoo

10. **Problem:** Mary Price Maddox laughs 7 times per class. If she teaches 4 classes a day for the 5 weekdays every week but doesn't laugh on Wednesdays, then how many times does she laugh after 5 weeks of teaching?

    Solution: Since Ms. Maddox does not laugh on Wednesdays, she laughs 4 days a week, 4 classes a day, 7 laughs per class. That is $4 \cdot 4 \cdot 7 = 112$ laughs per week. Then since it asks for 5 weeks of teaching, we multiply 112 by 5 to get $\boxed{560}$ laughs.

    Proposed by Eddie Wang

11. **Problem:** Let $ABCD$ be a unit square. If E is the midpoint of AB and F lies inside $ABCD$ such that CFD is an equilateral triangle, the positive difference between the area of CED and CFD can be expressed in the form $\frac{a-\sqrt{b}}{c}$, where a, b, c are in lowest simplified terms. What is $a + b + c$?

    Solution: Note that CED is a triangle with height 1 and base 1, so the area of CED is $\frac{1}{2}$. Further, CFD is an equilateral triangle with base 1, so its area is $\frac{1^2\sqrt{3}}{4} = \frac{\sqrt{3}}{4}$. Thus the positive difference between the area of CED and CFD is $\frac{2-\sqrt{3}}{4}$. Thus the answer is $2 + 3 + 4 = \boxed{9}$.

    Proposed by Anuj Sakarda

12. **Problem:** Eddie has musician's syndrome. Whenever a song is a C, A, or F minor, he begins to cry and his body becomes very stiff. On the other hand, if the song is in G minor, A flat major, or E flat major, his eyes open wide and he feels like the happiest human being ever alive. There are a total of

24 keys. How many different possibilities are there in which he cries [and doesn't feel happy] while playing one song with two distinct keys?

Solution: There are 24 keys in total: 3 that make Eddie happy, 3 that make Eddie sad, and 18 that do not incite any emotions in Eddie. Then there are two possibilities for when Eddie cries and doesn't feel happy while playing one song with two distinct keys: either Eddie plays a song with two sad keys, or one sad key and one neutral key. Eddie can choose $\binom{3}{2} = 3$ different pairings of sad and sad keys. Eddie can choose $\binom{18}{1} \cdot \binom{3}{1} = 54$ different pairings of sad and neutral keys. This gives us $54 + 3 = \boxed{57}$ possibilities in total.

Note: The problem in the test pdf did not include the phrase in the brackets, which changes the problem. So, we are also accepting $\boxed{66}$ as answer, which comes from $\binom{3}{2} + 3 \cdot 21$ because it includes the case of a sad and happy key.

Proposed by Eddie Wang

13. **Problem:** What positive integer must be added to both the numerator and denominator of $\frac{12}{40}$ to make a fraction that is equivalent to $\frac{4}{11}$?

Solution: Let this positive integer be x. Then $\frac{x+12}{x+40} = \frac{4}{11}$. After cross multiplying we have that

$$11x + 132 = 4x + 160,$$

so $x = \frac{28}{7} = \boxed{4}$.

Proposed by Aaron Zhang

14. **Problem:** The number 0 is written on the board. Each minute, Gene the genie either multiplies the number on the board by 3 or 9, each with equal probability, and then adds either 1,2, or 3, each with equal probability. Find the expected value of the number after 3 minutes.

Solution: Each minute, if the number on the board is x, then the next minute it becomes $3x+1, 3x+2, 3x+3, 9x+1, 9x+2, 9x+3$, which average to $6x+2$. By Linearity of Expectation, the expected value of the number on the board is $6(0)+2 = 2$ after 1 minute, $6(2)+2 = 14$ after 2 minutes, and $6(14)+2 = \boxed{86}$ after 3 minutes.

Proposed by Jerry Tan

15. **Problem:** x satisfies $\frac{1}{x+\frac{1}{1+\frac{1}{2}}} = \frac{1}{2+\frac{1}{1-\frac{1}{2+\frac{1}{2}}}}$ Find x.

Solution: First we simplify the left hand side:

$$\frac{1}{x+\frac{1}{1+\frac{1}{2}}} = \frac{1}{x+\frac{1}{\frac{3}{2}}} = \frac{1}{x+\frac{2}{3}} = \frac{1}{\frac{3x+2}{3}} = \frac{3}{3x+2}.$$

Then we simplify the right hand side:

$$\frac{1}{2+\frac{1}{1-\frac{1}{2+\frac{1}{2}}}} = \frac{1}{2+\frac{1}{1-\frac{1}{\frac{5}{2}}}} = \frac{1}{2+\frac{1}{1-\frac{2}{5}}} = \frac{1}{2+\frac{1}{\frac{3}{5}}} = \frac{1}{2+\frac{5}{3}} = \frac{3}{11}.$$

So then we have that $\frac{3}{3x+2} = \frac{3}{11}$, so

$$3x + 2 = 11.$$

Solving for x we get $x = \boxed{3}$.

Proposed by Aaron Zhang

16. **Problem:** How many different points in a coordinate plane can a bug end up on if the bug starts at the origin and moves one unit to the right, left, up or down every minute for 8 minutes?

Solution: Let the final point the bug ends on be (x, y). Notice that $|x| + |y|$ must be equal to either $8, 6, 4, 2$ or 0. For every case except 0, we can first count the number of lattice points where neither x nor y are 0. Take the case where $|x| + |y| = 8$. In the first quadrant, there are 7 lattice points that work. Notice that for each of these 7 points, there are 3 other corresponding points that work in the other quadrants. Thus, there are 28 lattice points, not counting ones on the two axes, that work. On the two axes, there are then 4 more points. Thus, for the case where the sum is 8, there are a total of 32 points that work. Repeating the process, we find that there are 24 points that work when the sum is 6, 16 points that work when the sum is 4, and 8 points that work when the sum is 2. Finally, only the origin works when the sum is 0. Thus, our answer is $32 + 24 + 16 + 8 + 1 = \boxed{81}$

Proposed by Anuj Sakarda

17. **Problem:** The triplets Addie, Allie, and Annie, are racing against the triplets Bobby, Billy, and Bonnie in a relay race on a track that is 100 feet long. The first person of each team must run around the entire track twice and tag the second person for the second person to start running. Then, the second person must run once around the entire track and tag the third person, and finally, the third person would only have to run around half the track. Addie and Bob run first; Allie and Billy second; Annie and Bonnie third. Addie, Allie, and Annie run at 50 feet per minute (ft/m), 25 ft/m, and 20 ft/m, respectively. If Bob, Billy, and Bonnie run half as fast as Addie, Allie, and Annie, respectively, then how many minutes will it take Bob, Billy, and Bonnie to finish the race. Assume that everyone runs at a constant rate.

Solution: Addie runs 200 feet, Allie runs 100 feet, and Annie runs 50 feet. It takes Addie 4 minutes, Allie 4 minutes, and Annie 2.5 minutes to run each of their segments, which makes a total of 10.5 minutes for the three to finish the race. Since Bob, Billy, and Bonnie each run half as fast, it will take them twice as long to finish the race or $\boxed{21}$ minutes.

Proposed by Annie Wang

18. **Problem:** James likes to play with Jane and Jason. If the probability that Jason and Jane play together is 1/3, while the probability that James and Jason is 1/4 and the probability that James and Jane play together is 1/5, then the probability that they all play together is $\frac{\sqrt{p}}{q}$ for positive integers p, q where p is not divisible by the square of any prime. Find $p + q$. Assume the events that they play are independent of each other.

Solution: Let x, y, z be the probability that James, Jane, Jason play, respectively. We have $yz = \frac{1}{3}, xz = \frac{1}{4}, xy = \frac{1}{5}$ and we want to find xyz. Multiplying the equations gives us $(xyz)^2 = \frac{1}{60}$ so then $xyz = \frac{\sqrt{15}}{30}$. The answer is $\boxed{45}$.

Proposed by Eddie Wang

19. **Problem:** Call an integer a *near-prime* if it is one more than a prime number. Find the sum of all *near-primes* less than 1000 that are perfect powers. (Note: a perfect power is an integer of the form n^k where $n, k \geq 2$ are integers.)

Solution: We have the equation $p + 1 = n^k$ for some prime p and integers $n, k \geq 2$. Rewrite as $n^k - 1 = p$. We can factor the LHS as $(n-1)(n^{k-1} + n^{k-2} + \cdots + 1)$. Thus we must have $1 = n - 1$ and $p = n^{k-1} + n^{k-2} + \cdots + 1$. Then $n = 2$, so we look at all powers of 2 greater than 2 and less than 1000, which are $4, 8, 16, 32, 64, 128, 256, 512$. Subtracting one from each, we get $3, 7, 31, 127$ are primes since $5|255$ and $7|511$. The sum is then $4 + 8 + 32 + 128 = \boxed{172}$.

Proposed by Jerry Tan

20. **Problem:** What is the integer solution to $\sqrt{\frac{2x-6}{x-11}} = \frac{3x-7}{x+6}$?

Solution: First we square both sides to obtain

$$\frac{2x-6}{x-11} = \frac{9x^2-42x+49}{x^2+12x+36}.$$

Then, after cross multiplying and bringing all terms to one side, we are left with the cubic

$$7x^3 - 159x^2 + 511x - 323 = 0.$$

Since x is an integer solution, $x|323$. Note that $323 = 17 \cdot 19$. After using synthetic division we find that $\boxed{19}$ is the answer.

Proposed by Aaron Zhang

21. **Problem:** Consider rectangle $ABCD$ with $AB = 12$ and $BC = 4$ with F, G trisecting DC so that F is closer to D. Then E is on AB. We call the intersection of EF and DB X, and the intersection of EG and DB is Y. If the area of $\triangle XYE$ is $\frac{8}{15}$, then what is the length of EB?

Solution: Let us call the length of EB x. Then note that $\triangle EXB$ is similar to $\triangle FXD$, with $EB : FD$ having a ratio of $x : 4$, and $\triangle EYB$ is similar to $\triangle GYD$, with $EB : GD$ having a ratio of $x : 8$. Then we can express the area of EXY as follows:

$$[EXY] = [EXB] - [EYB].$$

The altitude to EB for EYB has the value $\frac{x}{x+8} \cdot 4$, whereas the altitude to EB for EXB has the value $\frac{x}{x+4} \cdot 4$. Thus we have that $[EXY] = [EXB] - [EYB]$, which is equal to

$$\frac{x}{2} \cdot \frac{x}{x+4} \cdot 4 - \frac{x}{2} \cdot \frac{x}{x+8} \cdot 4 = 2x\left(\frac{x}{x+4} - \frac{x}{x+8}\right).$$

Combining the fractions and simplifying, we have that $[EXY] = \frac{8x^2}{x^2+12x+32} = \frac{8}{15}$ as given. Cross-multiplying and combining like terms, we have that $14x^2 - 12x - 32 = 0$. This factors into $2(x-2)(7x+8) = 0$, and the only positive solution is $x = 2$. Thus $EB = \boxed{2}$.

Proposed by Jerry Tan

22. **Problem:** The sum

$$\sum_{n=2} \frac{1}{4n^2-1}$$

can be expressed as a common fraction $\frac{a}{b}$ in lowest terms. Find $a+b$.

Solution: Begin by noticing that we can rewrite our sum as:

$$\sum_{n=2} \frac{1}{(2n+1)(2n-1)}$$

which can then be rewritten as:

$$\sum_{n=2} \frac{1}{2}(\frac{1}{(2n-1} - \frac{1}{2n+1})$$

$$= \frac{1}{2}[(\frac{1}{3} - \frac{1}{5}) + (\frac{1}{5} - \frac{1}{7}) + (\frac{1}{7} - \frac{1}{9}) + \ldots]$$

Notice that every term of the sum will cancel except for the first term, so the sum is simply equal to $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$. Our answer is $1 + 6 = \boxed{7}$

Proposed by Aaron Zhang

23. **Problem:** In square $ABCD$, M, N, O, P are points on sides $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DA}$, respectively. If $AB = 4$, $AM = BM$ and $DP = 3AP$, the least possible value of $MN + NO + OP$ can be expressed as $\sqrt{x}$ for some integer x. Find x.

    Solution: Reflect points O, P over line CD and then over line BC. Let the images be O', P' respectively. Then $MN + NO + OP = MN + NO' + O'P'$. By the triangle inequality we have $MN + NO' + O'P' \geq MP'$. We easily find $MP' = \sqrt{7^2 + 6^2} = \sqrt{85}$. The answer is $\boxed{85}$.

    Proposed by Justin Shan

24. **Problem:** Grand-Ovich the ant is at a vertex of a regular hexagon and he moves to one of the adjacent vertices every minute with equal probability. Let the probability that after 8 minutes he will have returned to the starting vertex at least once be the common fraction $\frac{a}{b}$ in lowest terms. What is $a + b$?

    Solution: We approach this problem through complementary counting. The only possible ways Grand-Ovich will not have returned to the initial vertex at least once are if either he moves a certain combination of 5 vertices in one direction and 3 in the other, or 6 vertices in one direction and 2 in the other.

    In the first case, there are 28 paths, and in the second case, there are 26 paths. In total, there are 54 paths that result in Grand-Ovich not returning to the original vertex. Every minute, Grand-Ovich has 2 choices, so after 8 minutes, there are $2^8 = 256$ possible paths for Grand-Ovich to take.

    Thus, the probability that Grand-Ovich will have returned to the initial vertex at least once is $\frac{256-54}{256} = \frac{202}{256} = \frac{101}{128}$. The answer is $\boxed{229}$.

    Proposed by Jerry Tan

25. **Problem:** Find the last two non-zero digits at the end of 2020! written it as a two digit number.

    Solution: First we find the largest power of 5 dividing 2020!, which is $404 + 80 + 16 + 3 = 503$. The problem wants us to find $\dfrac{2020!}{10^{503}} \pmod{100}$. We split this into finding mod 4 and mod 25. Start with mod 25.

    Let $v(n)$ be the largest power of 5 dividing $n!$ and let $X(n)$ equal $\dfrac{n!}{v(n)}$.

    Notice that $1 \cdot 2 \cdot 3 \cdot 4 \equiv -1 \pmod{25}$. In fact the product of any four consecutive integers none of which are divisible by 5 always is $-1 \pmod{25}$. Then $X(2020) \equiv (1 \cdot 2 \cdot 3 \cdot 4) \cdot (6 \cdot 7 \cdot 8 \cdot 9) \ldots (2016 \cdot 2017 \cdot 2018 \cdot 2019) \cdot X(404) \equiv X(404) \pmod{25}$. Continuing in this manner

$$\begin{aligned} X(404) &\equiv 401 \cdot 402 \cdot 403 \cdot 404 \cdot X(80) \\ &\equiv -X(80) \\ &\equiv -X(16) \\ &\equiv -1 \cdot 16 \cdot (-X(3)) \\ &\equiv 96 \pmod{25}. \end{aligned}$$

    Since 2 and 25 are relatively prime, we want $\dfrac{X(2020)}{2^{503}} \equiv \dfrac{96}{2^3} \equiv 12 \pmod{25}$ since $2^{20} \equiv 1 \pmod{25}$ by Euler's totients. $\dfrac{2020!}{10^{503}}$ is obviously $0 \pmod{4}$, so combining it must be $12 \pmod{100}$. Our answer is $\boxed{12}$.

    Proposed by Anuj Sakarda

Accuracy Round

1. **Problem:** James has 8 Instagram accounts, 3 Facebook accounts, 4 QQ accounts, and 3 YouTube accounts. If each Instagram account has 19 pictures, each Facebook account has 5 pictures and 9 videos, each QQ account has a total of 17 pictures, and each YouTube account has 13 videos and no pictures, how many pictures in total does James have in all these accounts?

   Solution: Note that since each Youtube account has no pictures, that information is completely unnecessary. Then from the Instagram accounts there are $8 \cdot 19 = 152$ pictures, from the Facebook accounts there are $3 \cdot 5 = 15$ pictures, and from the QQ accounts there are $4 \cdot 17 = 68$ pictures. So the total number of pictures James has is $152 + 15 + 68 = \boxed{235}$.

   Proposed by Poonam Sahoo

2. **Problem:** If Poonam can trade 7 *shanks* for 4 *shinks*, and she can trade 10 *shinks* for 17 *shenks*. How many *shenks* can Poonam get if she traded all of her 105 *shanks*?

   Solution: We have that 105 shanks corresponds to $\frac{105}{7} \cdot 4 = 60$ shinks. Then 60 shinks corresponds to $\frac{60}{10} \cdot 17 = \boxed{102 \text{ shenks}}$.

   Proposed by Justin Shan

3. **Problem:** Jerry has a bag with 3 red marbles, 5 blue marbles and 2 white marbles. If Jerry randomly picks two marbles from the bag without replacement, the probability that he gets two different colors can be expressed as a fraction $\frac{m}{n}$ in lowest terms. What is $m + n$?

   Solution: Let's use complimentary counting and figure out the probability of Jerry getting two marbles of the same color. There are 10 marbles in total, so there are $\binom{10}{2} = 45$ ways to choose two marbles from the bag without replacement. Further there are $\binom{3}{2} = 3$, $\binom{5}{2} = 10$, $\binom{2}{2} = 1$ ways to choose two of the same color for red, blue, white respectively. Thus the probability of Jerry getting two marbles of the same color is

   $$\frac{3 + 10 + 1}{45} = \frac{14}{45}.$$

   Thus the probability of getting two marbles of a different color is $1 - \frac{14}{45} = \frac{31}{45}$. Thus $m = 31, n = 45$, so the answer is $m + n = 31 + 45 = \boxed{76}$.

   Proposed by Nithin Kavi

4. **Problem:** Bob's favorite number is between 1200 and 4000, divisible by 5, has the same units and hundreds digits, and the same tens and thousands digits. If his favorite number is even and not divisible by 3, what is his favorite number?

   Solution: Bob's favorite number is of the form $BABA$ where A must be even. Further, since Bob's favorite number is divisible by 5, then $A = 0$. Since Bob's favorite number is between 1200 and 4000, the choices for B are 2 and 3 (it cannot be 1 because 1010 is less than 1200). Further, Bob's favorite number is not divisible by 3, which means that the sum of digits $B + 0 + B + 0 = 2B$ is not divisible by 3, so therefore $B = 2$. Thus Bob's favorite number is $\boxed{2020}$!

   Proposed by Jerry Tan

5. **Problem:** Consider a unit cube $ABCDEFGH$. Let O be the center of the face $EFGH$. The length of $\overline{BO}$ can be expressed in the form $\frac{\sqrt{a}}{b}$, where a and b are simplified to lowest terms. What is $a + b$?

   Solution: Notice that BOF is a right triangle with $\overline{BO}$ as the hypotenuse. So now we try to find $\overline{BO}$ using the Pythagorean theorem. Note that since $ABCDEFGH$ is a unit cube, $\overline{BF} = 1$. Further, $\overline{OF}$ is just half of the face diagonal $\overline{HF}$, so $\overline{OF} = \frac{\sqrt{2}}{2}$. Thus we know that

$$BO = \sqrt{1^2 + \frac{2}{4}} = \sqrt{\frac{6}{4}} = \frac{\sqrt{6}}{2}.$$

Thus $a = 6, b = 2$, so $a + b = \boxed{8}$.

Proposed by Annie Wang

6. **Problem:** Mr. Eddie Wang is a crazy rich boss who owns a giant company in Singapore. Even though Mr. Wang appears friendly, he finds great joy in firing his employees. His immediately fires them when they say "hello" and/or "goodbye" to him. It is well known that 1/2 of the total people say "hello" and/or "goodbye" to him everyday. If Mr. Wang had 2050 employees at the end of yesterday, and he hires 2 new employees at the beginning of each day, in how many days will Mr. Wang first only have 6 employees left?

   Solution: We make a list keeping track of the number of employees each day. Starting at 2050, we add 2 and divide by 2 for each day. We get $2050, 1026, 514, 258, 130, 66, 34, 18, 10, 6$. Since we started from yesterday, he will have 6 employees in $\boxed{8}$ days. We are also accepting $\boxed{9}$ as an answer.

   Proposed by Annie Wang

7. **Problem:** In $\triangle ABC$, $AB = 5, AC = 6$. Let D, E, F be the midpoints of $\overline{BC}, \overline{AC}, \overline{AB}$, respectively. Let X be the foot of the altitude from D to $\overline{EF}$. Let $\overline{AX}$ intersect $\overline{BC}$ at Y. Given $DY = 1$, the length of BC is $\frac{p}{q}$ for relatively prime positive integers p, q. Find $p + q$.

   Solution: Let G be the foot of altitude from A to BC. Triangles $\triangle YXD$ and $\triangle YAG$ are similar with ratio 1:2. Thus $GD = 1$. Let $x = BG$. Since D is a midpoint, $YC = BG = x$. Then Pythagorean on ABG, AGC gives us $AG^2 = 25 - x^2 = 36 - (x+2)^2$ so $4x + 4 = 11$ and $x = \frac{7}{4}$. Then $BC = 2x + 2 = \frac{11}{2}$ so the answer is $\boxed{13}$.

   Proposed by Jerry Tan

8. **Problem:** Given $\frac{1}{2006} = \frac{1}{a} + \frac{1}{b}$ where a is a 4 digit positive integer and b is a 6 digit positive integer, find the smallest possible value of b.

   Solution: We multiply by $2006ab$ on both sides to obatin $ab = 2006a + 2006b$ or $ab - 2006a - 2006b = 0$. We can factor $(a - 2006)(b - 2006) = 2006^2$. Note that $2006 = 2 \cdot 17 \cdot 59$. We know that a, b are positive. Since a is four digits $a - 2006$ is small, so we look at small factors of 2006^2. The smallest factors are $1, 2, 4, 17, 34, 59$. For these values of $a - 2006$, only 17 and 34 give us a 6-digit number for b. Then the smaller value of b is $\frac{2006^2}{34} + 2006 = 59 \cdot 2006 + 2006 = 60 \cdot 2006 = \boxed{120360}$.

   Proposed by Aaron Zhang

9. **Problem:** Pocky the postman has unlimited stamps worth $5, 6$ and 7 cents. However, his post office has two very *odd* requirements: On each envelope, an odd number of 7 cent stamps must be used, and the total number of stamps used must also be odd. What is the largest amount of postage money Pocky cannot make with his stamps, in cents?

   Solution: Denote x, y, z the number of $5, 6, 7$ cent stamps used, respectively. We look at cases based on z. If $z = 1$, then varying x, y over $x + y = 8$ gives postages in the interval $[47, 55]$. For $x + y = 6$ we get $[37, 43]$. For $x + y = 4$ we get $[27, 31]$.

   Now for $z = 3$, varying x, y for $x + y = 2$ gives $[31, 33]$ and $x + y = 4$ gives $[41, 45]$.

   Note that if N can be made with stamps, so can $N + 10$ by adding two 5-cent stamps. Since $46 = 1(5) + 1(6) + 5(7)$ the numbers from 46 to 55 are all possible, so all numbers ≥ 56 can be made as well.

The largest remaining number not in any of the above intervals is 36. Since 36 does not lie in any interval for $z = 1$ or $z = 3$, we look at $z \geq 5$. But $z \geq 5$ obviously doesn't work, so the answer is $\boxed{36}$.

Proposed by Jerry Tan

10. **Problem:** Let $ABCDEF$ be a regular hexagon with side length 2. Let G be the midpoint of side DE. Now let O be the intersection of BG and CF. The radius of the circle inscribed in triangle BOC can be expressed in the form $\frac{a\sqrt{b}-\sqrt{c}}{d}$, where a, b, c, d are simplified to lowest terms. What is $a + b + c + d$?

    Solution: We will use the area formulas regarding triangles to find the radius of the incircle of BOC. We already know from the problem that BC is 2. Since G is the midpoint of DE, we have that $GD = 1$. Note that FC is parallel to ED and also is the perpendicular bisector of BD. Let us call the intersection of BD and FC X. Then BOX is similar to BGD with $BX = \frac{1}{2}BD$. Note that since BXC is a $30-60-90$ triangle, $BX = \sqrt{3}$ and $XC = 1$. Then $BD = 2\sqrt{3}$, so we can solve for BG using the Pythagorean theorem:

$$BG = \sqrt{(2\sqrt{3})^2 + 1} = \sqrt{13}.$$

    By the similarity we discussed earlier, $BO = \frac{\sqrt{13}}{2}$ and $BX = \frac{1}{2}$. Thus we have found all of the side-lengths of BOC : $BC = 2$, $OC = 1 + \frac{1}{2} = \frac{3}{2}$, and $BO = \frac{\sqrt{13}}{2}$. Now we find the area of BOC. As mentioned earlier, $BX = \sqrt{3}$ and is the height of BOC since it is perpendicular to OC. Thus the area of

$$[BOC] = \frac{1}{2} \cdot \frac{3}{2} \cdot \sqrt{3} = \frac{3\sqrt{3}}{4}.$$

    Let us call the radius of the incircle of BOC r. Then we have that

$$\frac{r\left(\frac{\sqrt{13}}{2} + 2 + \frac{3}{2}\right)}{2} = \frac{3\sqrt{3}}{4}.$$

    Thus,

$$r = \frac{\frac{3\sqrt{3}}{2}}{\frac{\sqrt{13}}{2} + \frac{7}{2}} = \frac{3\sqrt{3}}{7 + \sqrt{13}}.$$

    Rationalizing the denominator and expressing in simplest terms, we have that

$$r = \frac{3\sqrt{3}(7 - \sqrt{13})}{36} = \frac{7\sqrt{3} - \sqrt{39}}{12}.$$

    Then $a = 7, b = 3, c = 39, d = 12$, so $a + b + c + d = \boxed{61}$.

    Proposed by Aaron Zhang

11. **Problem:** **Estimation:** What is the total number of characters in all of the participants' email addresses in the Accuracy Round?

    Solution: There are $\boxed{10879}$ characters in total in the email addresses of all the participants in the Accuracy round.

    Proposed by Aaron Zhang

Team Round

Round 1

1. **Problem:** A person asks for help every 3 seconds. Over a time period of 5 minutes, how many times will they ask for help?

   Solution: 5 minutes is equivalent to 300 seconds, so the person will ask for help $\frac{300}{3} = \boxed{100}$ times.

   Proposed by Anuj Sakarda

2. **Problem:** In a big bag, there are 14 red marbles, 15 blue marbles, and 16 white marbles. If Anuj takes a marble out of the bag each time without replacement, how many marbles does Anuj need to remove to be sure that he will have at least 3 red marbles?

   Solution: For Anuj to be sure, he must remove an amount of marbles equivalent to the sum of 3 red marbles and all the blue and white marbles, which is $3 + 15 + 16 = \boxed{34}$

   Proposed by Aaron Zhang

3. **Problem:** If Josh has 5 distinct candies, how many ways can he pick 3 of them to eat?

   Solution: The answer is simply $\binom{5}{3} = \boxed{10}$ ways. Another way to see this is that he can choose $5 \cdot 4 \cdot 3$ combinations for the first, second, third candies, and then divide by $3 \cdot 2 \cdot 1$ because order does not matter.

   Proposed by Aaron Zhang

Round 2

1. **Problem:** Annie has a circular pizza. She makes 4 straight cuts. What is the minimum number of slices of pizza that she can make?

   Solution: The minimum number of slices occurs if Annie makes 4 cuts that are parallel to one another, making a total of $\boxed{5}$ slices.

   Proposed by Aaron Zhang

2. **Problem:** What is the sum of the first 4 prime numbers that can be written as the sum of two perfect squares?

   Solution: We know that $2 = 1^2 + 1^2$, 3 does not work, $5 = 1^2 + 2^2$, 7 does not work, 11 does not work, $13 = 2^2 + 3^2$, and $17 = 4^2 + 1^2$. Thus the sum is $2 + 5 + 13 + 17 = \boxed{37}$.

   Proposed by Justin Shan

3. **Problem:** Consider a regular octagon ABCDEFGH inscribed in a circle of area 64π. If the length of arc ABC is $n\pi$, what is n?

   Solution: Since the area of the circle is 64π, the radius of the circle is 8, and thus the circumference of the circle must be 16π. The 8 vertices of the regular octagon split the circumference of the circle into 8 equivalent pieces, so arc ABC is simply $\frac{2}{8}$ the circumference of the circle or 4π. Thus, the answer is $\boxed{4}$

   Proposed by Aaron Zhang

Round 3

1. **Problem:** Let ABCDEF be an equiangular hexagon with consecutive sides of length 6, 5, 3, 8, and 3. Find the length of the sixth side.

Solution: An equiangular hexagon can be created from the intersection of two equilateral triangles. Let the last side of the equiangular hexagon be x. To solve for x, we can set the lengths of two sides of one of the big equilateral triangles to be equivalent. We get $x + 6 + 5 = 5 + 3 + 8$. Thus, $x = \boxed{5}$

Proposed by Anuj Sakarda

2. **Problem:** Jack writes all of the integers from 1 to n on a blackboard except the even primes. He selects one of the numbers and erases all of its digits except the leftmost one. He adds up the new list of numbers and finds that the sum is 2020. What was the number he chose?

   Solution: Note that the sume of all the numbers from 1 to n "except even primes" is just $\frac{n(n+1)}{2} - 2$. This is closest to $\binom{65}{2} - 2 = 2080 - 2 = 2078$. So suppose we subtract a number of the form $10A + B$ where A, B are its digits then add in A again since it is the left most digit. Then we have

   $$2078 - (10A + B) + A = 2020,$$

   so

   $$58 = 9A + B.$$

   Solving this equation we have $A = 6, B = 4$, so the number that was chosen was $\boxed{64}$.

   Proposed by Anuj Sakarda

3. **Problem:** Our original competition date was scheduled for April 11, 2020 which is a Saturday. The numbers 4116 and 2020 have the same remainder when divided by x. If x is a prime number, find the sum of all possible x.

   Solution: If $4116 \equiv 2020 \pmod{x}$ that means that $x|(4116 - 2020)$, so $x|2096$. Note that $2096 = 2^4 \cdot 131$, so its only prime factors are 2 and 131. Thus the sum of all such possible x is $2 + 131 = \boxed{133}$.

   Proposed by Aaron Zhang

Round 4

1. **Problem:** The polynomials $5p^2 + 13pq + cq^2$ and $5p^2 + 13pq - cq^2$ where c is a positive integer can both be factored into linear binomials with integer coefficients. Find c.

   Solution: Let us write $5p^2 + 13pq + cq^2 = (5p + Yq)(p + Xq) = 5p^2 + (5X + Y)pq + XYq^2$. Then we have that $5X + Y = 13$, which has solutions $(1, 8)$ and $(2, 3)$. When we try $(2, 3)$, we have that $c = 6$. Lo and behold, $5p^2 + 13pq - 6q^2 = (5p - 2q)(p + 3q)$, so $c = \boxed{6}$ works.

   Proposed by Aaron Zhang

2. **Problem:** In a Cartesian coordinate plane, how many ways are there to get from $(0, 0)$ to $(2, 3)$ in 7 moves, if each move consists of a moving one unit either up, down, left, or right?

   Solution: Denote the moves U, D, L, R. We need 2 R's and 3 U's as well as a pair of RL or a pair of UD.

   Case 1: Extra pair is RL.

   Then we have 3 R's, 3 U's and 1 L to arrange, which has $\dfrac{7!}{3!3!1!} = 140$ ways.

   Case 2: Extra pair is UD.

   We have 2 R's, 4U's, 1 D for a total of $\dfrac{7!}{2!4!1!} = 105$ ways. The total is then $140 + 105 = \boxed{245}$.

   Proposed by Justin Shan

3. **Problem:** Bob the Builder is building houses. On Monday he finds an empty field. Each day starting on Monday, he finishes building a house at noon. On the n^{th} day, there is a $\frac{n}{8}$ chance that a storm will appear at 3:14 PM and destroy all the houses on the field. At any given moment, Bob feels sad if and

only if there is exactly 1 house left on the field that is not destroyed. The probability that he will not be sad on Friday at 6PM can be expressed as $\frac{p}{q}$ in simplest form. Find $p+q$.

Solution: We use complementary counting, so we want to find the probability that Bob will be sad. A storm on Friday would result in no houses being left, and no storm on Thursday means there will be more than 1 house left. Thus, the only way Bob will be sad is if there is a storm on Thursday and no storm on Friday. The chance of this is $\frac{4}{8} \cdot \frac{3}{8} = \frac{3}{16}$ so the complement is $\frac{13}{16}$. The answer is $\boxed{29}$.

Proposed by Jerry Tan

Round 5

1. **Problem:** Quadrilateral ABCD is such that $\angle ABC = \angle ADC = 90°$, $\angle BAD = 150°$, $AD = 3$, and $AB = \sqrt{3}$. The area of ABCD can be expressed as $p\sqrt{q}$ for positive integers p, q where q is not divisible by the square of any prime. Find $p+q$.

   Solution: Extend DA and CB to meet at E. Then $\triangle ABE$ is a 30-60-90. We have $AE = \frac{2}{\sqrt{3}}AB = 2$. Since $\triangle EDC$ is also 30-60-90, we have $DC = DE\sqrt{3} = 5\sqrt{3}$. Then the area of $ABCD$ is $\frac{5(5\sqrt{3})}{2} - \frac{\sqrt{3}}{2} = 12\sqrt{3}$. The answer is $\boxed{15}$.

   Proposed by Jerry Tan

2. **Problem:** Neetin wants to gamble, so his friend Akshay describes a game to him. The game will consist of three dice: a 100-sided one with the numbers 1 to 100, a tetrahedral one with the numbers 1 to 4, and a normal 6-sided die. If Neetin rolls numbers with a product that is divisible by 21, he wins. Otherwise, he pays Akshay 100 dollars. The number of dollars that Akshay must pay Neetin for a win in order to make this game fair is $\frac{a}{b}$ for relatively prime positive integers a, b. Find $a+b$. (Fair means the expected net gain is \$0.)

   Solution: We first have to find the probability that Neetin wins. Since $21 = 3 \cdot 7$, Neetin must roll a multiple of 7, which must come from the 100 die. There are 14 multiples of 7 less than 100, 4 of which are multiples of 21.

   Case 1: Rolling a multiple of 7 that is not a multiple of 21 on the 100-die. There is a $\frac{10}{100}$ chance of this. Then the product of the other two die must be a multiple of three. The chance of this not happening is $\frac{3}{4} \cdot \frac{4}{6} = \frac{1}{2}$, thus the complement is $\frac{1}{2}$. This case has an over all $\frac{10}{200} = \frac{1}{20}$ chance of success for Neetin.

   Case 2: Rolling a multiple of 21 on the 100-die. There is a $\frac{4}{100}$ chance of this, and the other two die can roll anything. Thus $\frac{1}{25}$ for this case.

   Neetin's chance of winning is then $\frac{1}{20} + \frac{1}{25} = \frac{9}{100}$.

   Let x be the number of dollars Akshay pays Neetin for a win. Neetin's expected net gain for this gamble is then $\frac{9}{100} \cdot x - \frac{91}{100} \cdot 100$ which must be 0. Then

   $$x = \frac{100}{9} \cdot \frac{9100}{100} = \frac{9100}{9},$$

   so the answer is $\boxed{9109}$.

   Proposed by Aaron Zhang

3. **Problem:** What is the sum of the absolute value of the fourth powers of the roots of the polynomial $P(x) = x^2 + 2x + 3$?

   Solution: Let r, s be the roots of $P(x)$. Then $P(r) = 0$ so

   $$r^2 = -2r - 3$$

   Then

   $$r^3 = -2r^2 - 3r = 4r + 6 - 3r = r + 6$$

   and

   $$r^4 = r^2 + 6r = 4r - 3.$$

   Thus $r^4 + s^4 = 4(r+s) - 6 = 4(-2) - 6 = -14$ using Vieta's. Our answer is $\boxed{14}$

   Proposed by Nithin Kavi

Round 6

1. **Problem:** Consider the set $\mathbb{S} = \{1, 2, 3, 4 \ldots 25\}$. How many ordered n-tuples $S_1 = (a_1, a_2, a_3 \ldots a_n)$ of pairwise distinct a_i exist such that $a_i \in \mathbb{S}$ and $i^2 \mid a_i$ for all $1 \le i \le n$?

   Solution: Since $25 = 5^2$ we know n is most 5. Now we consider 5 cases on the value of n. In each case we start with the largest i when choosing a_i.

   Case 1: $n = 1$. 25 choices.

   Case 2: $n = 2$. 6 choices for a_2, then 24 choices for a_1. Total is $6 \cdot 24 = 144$.

   Case 3: $n = 3$. 2 choices for a_3, then 6 choices of a_2 and 23 choices of a_1. Total is $2 \cdot 6 \cdot 23 = 276$.

   Case 4: $n = 4$. Any choice of a_4 reduces the choices for a_2 by 1. a_4 must be 16, then we have 2 choices of a_3 and 5 choices of a_2, 22 choices of a_1. Total is $1 \cdot 2 \cdot 5 \cdot 22 = 220$.

   Case 5: $n = 5$. a_5 must be 25, which reduces a choice of a_1. So the total is $2 \cdot 5 \cdot 21 = 210$.

   The grand total is $25 + 144 + 276 + 220 + 210 = \boxed{875}$.

   Proposed by Jerry Tan

2. **Problem:** How many ways are there to place 2 identical rooks and 1 queen on a $4x4$ chessboard such that no piece attacks another piece? (A queen can move diagonally, vertically or horizontally and a rook can move vertically or horizontally)

   Solution: We do casework on the queen position. We split the board into 4 corner squares, 4 center squares, and 8 "edge" squares.

   Case 1: Queen is on a corner. There are 6 squares not attacked by the queen, and after placing one rook there will be 3 squares for the second rook. There $4 \cdot 6 \cdot 3/2 = 36$ ways in this case.

   Case 2: Queen in center. There are 4 squares not attacked by the queen, and after placing one rook there will be 2 squares left. There are $4 \cdot 4 \cdot 2/2 = 16$ ways in this case.

   Case 3: Queen on an edge square. There are 6 not attacked squares, in three rows of 1,2,3 squares over 3 columns. If a rook is placed in the row with 1 square, there are 4 choices for the second rook. In the row with 2 squares, there are 3 choices and in the row with 3 square there are 2 choices. The total is $\dfrac{1 \cdot 4 + 2 \cdot 3 + 3 \cdot 2}{2} = 8$ for any given queen placement. There are 8 edge squares, so $8 \cdot 8 = 64$ total.

   Adding up all cases we get $36 + 16 + 64 = \boxed{116}$.

   Proposed by Jerry Tan

3. **Problem:** Let L be an ordered list $l_1, l_2, \ldots, l_{36}$ of consecutive positive integers who all have the sum of their digits not divisible by 11. It is given that l_1 is the least element of L. Find the least possible value of l_1.

Solution: Let $s(n)$ be the sum of the digits of n. For any k, the numbers $s(10k)$ through $s(10k+9)$ cover 10 distinct residues mod 11. Since L has 36 elements, L must contain $s(10m)$ through $s(10m+19)$ for some m. Then $s(10m) \equiv s(10(m+1)) \equiv 1 \pmod{11}$, otherwise some sum of digits is divisible by 11.

If j is the number of consecutive 9's at the end of the decimal representation of $10m+9$, then we have that $s(10m+10) - s(10m+9) = -(8+9(j-1)) = 1-9j$ which we want to equal $1-10 = -9 \pmod{11}$. The smallest value of j that works is $j=6$, which points to 999999. We check that the numbers between 999990 and 1000009 do satisfy the problem conditions. Since $s(999980) = 44$, we can take $l_1 = 999981$ and $l_{36} = 1000016$. Thus the answer is $\boxed{999981}$.

Proposed by Justin Shan

Round 7

1. **Problem:** Spencer, Candice, and Heather love to play cards, but they especially love the highest cards in the deck - the face cards (jacks, queens, and kings). They also each have a unique favorite suit: Spencer's favorite suit is spades, Candice's favorite suit is clubs, and Heather's favorite suit is hearts. A dealer pulls out the 9 face cards from every suit except the diamonds and wants to deal them out to the 3 friends. How many ways can he do this so that none of the 3 friends will see a single card that is part of their favorite suit?

   Solution: Notice that the suits of the 3 cards that each person has is determined when the suits of the 3 cards of only 1 person is determined (i.e. determining the suits of the cards that Spencer has will determine the suits of the cards of both Candice and Heather). Without loss of generality, we will use use Spencer to determine the suits of the cards of every person.

   There are, thus, 4 cases that we need to consider (2 of which are equivalent):

   Case 1 (Spencer has 3 hearts and no clubs): There is only one way the dealer can do this (i.e. if Spencer gets all 3 hearts, Candice gets all 3 clubs, and Heather gets all 3 hearts.

   Case 2 (Spencer has 2 hearts and 1 club): This implies that Candice has 2 spades and 1 heart, and Heather has 2 clubs and 1 spade. Notice that we only need to determine the value of Spencer's club, Candice's heart, and Heather's spade to determine every hand. There are $3 \times 3 \times 3$ ways for the dealer to deal the cards out this way for a total of 27 ways for this case.

   Case 3 (Spencer has 1 heart and 2 clubs): This case is equivalent to case 2. There are 27 ways for this case.

   Case 4 (Spencer has no hearts and 3 clubs): This case is equivalent to case 1. There is only one way for this case.

   The answer is $1+27+27+1 = \boxed{56}$

   Proposed by Aaron Zhang

2. **Problem:** Suppose a sequence of integers satisfy the recurrence $a_{n+3} = 7a_{n+2} - 14a_{n+1} + 8a_n$. If $a_0 = 4$, $a_1 = 9$, and $a_2 = 25$, find a_{16}. Your answer will be in the form $2^a + 2^b + c$, where $2^a < a_{16} < 2^{a+1}$ and b is as large as possible. Find $a+b+c$.

   Solution: The characteristic polynomial of the recurrence is $x^3 = 7x^2 - 14x + 8$ or

   $$x^3 - 7x^2 + 14x - 8 = 0.$$

   This factors as $(x-4)(x-2)(x-1) = 0$. Since the roots are $1, 2, 4$, we know

   $$a_n = c_1 4^n + c_2 2^n + c_3 1^n$$

for some constants c_1, c_2, c_3. We can plug in $n = 0, 1, 2$ to get

$$\begin{aligned} 4 &= c_1 + c_2 + c_3 \\ 9 &= 4c_1 + 2c_2 + c_3 \\ 25 &= 16c_1 + 4c_2 + c_3 \end{aligned}$$

Solving we get $c_1, c_2, c_3 = 1, 2, 1$. Thus $a_n = 4^n + 2^{n+1} + 1$, so $a_{16} = 4^{16} + 2^{17} + 1 = 2^{32} + 2^{17} + 1$. The answer is $32 + 17 + 1 = \boxed{50}$.

Proposed by Anuj Sakarda

3. **Problem:** Parallel lines l_1 and l_2 are 1 unit apart. Unit square $WXYZ$ lies in the same plane with vertex W on l_1. Line l_2 intersects segments YX and YZ at points U and O, respectively. Given $UO = \frac{9}{10}$, the inradius of $\triangle YOU$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n. Find $m + n$.

   Solution: We need to find the area and perimeter of $\triangle YOU$. To start, Let F be the foot of the altitude from W to UO. We are given by the problem that $WX = WF = WZ = 1$. Thus W is the circumcircle of $\triangle XFZ$. Then UX, UF are tangents to the circumcircle, so $UX = UF$. Similarly $OF = OZ$. Thus the perimeter of $\triangle YOU$ is

$$\begin{aligned} UO + YU + YO &= UO + 1 - XU + 1 - ZO \\ &= 2 + UO - (UF + OF) \\ &= 2. \end{aligned}$$

   Now we try to find the area. We have that

$$YU^2 + YO^2 = UO^2 \ (i)$$

   and

$$YU + YO = 2 - UO \ (ii)$$

   and the area we area looking for is $(YO)(YU)/2$. We take the square of (ii) and subtract (i) to get $2(YU)(YO) = 4 - 4(UO)$ so $(YU)(YO)/2 = 1 - UO = \frac{1}{10}$. Finally, $r = \frac{A}{s} = \frac{\frac{1}{10}}{1} = \frac{1}{10}$. Then $m + n = \boxed{11}$.

   Proposed by Jerry Tan

Round 8

1. **Problem:** Let A be the number of contestants who participated in at least one of the three rounds of the 2020 ABMC April contest. Let B be the number of times the letter b appears in the Accuracy Round. Let M be the number of people who submitted both the speed and accuracy rounds before 2:00 PM EST. Further, let C be the number of times the letter c appears in the Speed Round. Estimate

$$A \cdot B + M \cdot C.$$

   Solution: There were a total of 568 people who participated in this year's ABMC across all rounds, so $A = 568$. The letter b appears 69 times in the Accuracy round, so $B = 69$. The number of people who submitted both speed and accuracy before 2:00 PM EST is 164, so $M = 164$. The letter c appears 108 times in the Speed Round, so $C = 108$.

   Thus, the answer is $568 \cdot 69 + 164 \cdot 108 = \boxed{56904}$

   Proposed by Jerry Tan