Acton-Boxborough Math Competition Online Contest

Friday, December 20 — Sunday, December 22, 2019

Contest Rules and Format

The 2019 December Contest consists of 10 problems — each with an answer between 0 and 100,000. The contest window is

Friday, December 20 to Sunday, December 22.

The following rules supersede any others regarding the Online Contests.

- To submit your responses, fill out a form on our website, `abmathcompetition.org`. Include your email, name, school, grade (we require this information for prize purposes).
- Submissions close at 10:00PM Eastern Standard Time, on the final day of the round.
- Computational aids such as calculators, abaci, slide rules, etc. are prohibited. Drawing aids such as protractors and rules are permissible, but computer software such as GeoGebra or Desmos are prohibited.
- You may only work individually on the problems—consultation with others is not permitted.
- You may take as much time as you wish during the contest window.
- Each problem is worth 1 point.
- Ties will be broken by the "most difficult" problem solved. If problem A is solved by a contestants, and problem B is solved by b contestants, with $a < b$, then problem A is more difficult than B.

Awards and Prizes

- Top scoring contestants within each round will have their names posted on our website.
- Scores will be emailed to contestants within one week of the contest's end.
- Top scoring contestants across all three rounds will receive prizes.
- Physical prizes (such as AoPS gift certificates, merchandise, calculators, etc.) will be given to top scorers if they attend the ABMC onsite contest.

Good luck!

Problems

1. Let a be an integer. How many fractions $\frac{a}{100}$ are greater than $\frac{1}{7}$ and less than $\frac{1}{3}$?.
2. Justin Bieber invited Justin Timberlake and Justin Shan to eat sushi. There were 5 different kinds of fish, 3 different rice colors, and 11 different sauces. Justin Shan insisted on a spicy sauce. If the probability of a sushi combination that pleased Justin Shan is 6/11, then how many non-spicy sauces were there?
3. A palindrome is any number that reads the same forward and backward (for example, 99 and 50505 are palindromes but 2020 is not). Find the sum of all three-digit palindromes whose tens digit is 5.
4. Isaac is given an online quiz for his chemistry class in which he gets multiple tries. The quiz has 64 multiple choice questions with 4 choices each. For each of his previous attempts, the computer displays Isaac's answer to that question and whether it was correct or not. Given that Isaac is too lazy to actually read the questions, the maximum number of times he needs to attempt the quiz to guarantee a 100% can be expressed as 2^{2^k}. Find k.
5. Consider a three-way Venn Diagram composed of three circles of radius 1. The area of the entire Venn Diagram is of the form $\dfrac{a}{b}\pi+\sqrt{c}$ for positive integers a, b, c where a, b are relatively prime. Find $a+b+c$. (Each of the circles passes through the center of the other two circles)
6. The sum of two four-digit numbers is 11044. None of the digits are repeated and none of the digits are 0s. Eight of the digits from 1-9 are represented in these two numbers. Which one is not?
7. Al wants to buy cookies. He can buy cookies in packs of 13, 15, or 17. What is the maximum number of cookies he can not buy if he must buy a whole number of packs of each size?
8. Let $\triangle ABC$ be a right triangle with base $AB = 2$ and hypotenuse $AC = 4$ and let AD be a median of $\triangle ABC$. Now, let BE be an altitude in $\triangle ABD$ and let DF be an altitude in $\triangle ADC$. The quantity $(BE)^2 - (DF)^2$ can be expressed as a common fraction $\frac{a}{b}$ in lowest terms. Find $a + b$.
9. Let $P(x)$ be a monic cubic polynomial with roots r, s, t, where t is real. Suppose that $r + s + 2t = 8, 2rs + rt + st = 12$ and $rst = 9$. Find $|P(2)|$.
10. Let S be the set $\{1, 2, ..., 21\}$. How many 11-element subsets T of S are there such that there does not exist two distinct elements of T such that one divides the other?