Acton-Boxborough Math Competition Online Contest

Saturday, October 19 — Sunday, October 20, 2019

Contest Rules and Format

The 2019 October Contest consists of 10 problems — each with an answer between 0 and 100,000. The contest window is

Saturday, October 19 to Sunday, October 20.

The following rules supersede any others regarding the Online Contests.

- To submit your responses, fill out a form on our website, `abmathcompetition.org`. Include your email, name, school, grade (we require this information for prize purposes).
- Submissions close at 10:00PM Eastern Standard Time, on the final day of the round.
- Computational aids such as calculators, abaci, slide rules, etc. are prohibited. Drawing aids such as protractors and rules are permissible, but computer software such as GeoGebra or Desmos are prohibited.
- You may only work individually on the problems—consultation with others is not permitted.
- You may take as much time as you wish during the contest window.
- Each problem is worth 1 point.
- Ties will be broken by the "most difficult" problem solved. If problem A is solved by a contestants, and problem B is solved by b contestants, with $a < b$, then problem A is more difficult than B.

Awards and Prizes

- Top scoring contestants within each round will have their names posted on our website.
- Scores will be emailed to contestants within one week of the contest's end.
- Top scoring contestants across all three rounds will receive prizes.
- Physical prizes (such as AoPS gift certificates, merchandise, calculators, etc.) will be given to top scorers if they attend the ABMC onsite contest.

Good luck!

Problems

1. Fluffy the Dog is an extremely fluffy dog. Because of his extreme fluffiness, children always love petting Fluffy anywhere. Given that Fluffy likes being petted 1/4 of the time, out of 120 random people who each pet Fluffy once, what is the expected number of times Fluffy will enjoy being petted?

2. Andy thinks of four numbers 27, 81, 36, and 41 and whispers the numbers to his classmate Cynthia. For each number she hears, Cynthia writes down every factor of that number on the whiteboard. What is the sum of all the different numbers that are on the whiteboard? (Don't include the same number in your sum more than once)

3. Charles wants to increase the area his square garden in his backyard. He increases the length of his garden by 2 and increases the width of his garden by 3. If the new area of his garden is 182, then what was the original area of his garden?

4. Antonio is trying to arrange his flute ensemble into an array. However, when he arranges his players into rows of 6, there are 2 flute players left over. When he arranges his players into rows of 13, there are 10 flute players left over. What is the smallest possible number of flute players in his ensemble such that this number has three prime factors?

5. On the AMC 9 (Acton Math Competition 9), 5 points are given for a correct answer, 2 points are given for a blank answer and 0 points are given for an incorrect answer. How many possible scores are there on the AMC 9, a 15 problem contest?

6. Charlie Puth produced three albums this year in the form of CD's. One CD was circular, the second CD was in the shape of a square, and the final one was in the shape of a regular hexagon. When his producer circumscribed a circle around each shape, he noticed that each time, the circumscribed circle had a radius of 10. The total area occupied by 1 of each of the different types of CDs can be expressed in the form $a + b\pi + c\sqrt{d}$ where d is not divisible by the square of any prime. Find $a + b + c + d$.

7. You are picking blueberries and strawberries to bring home. Each bushel of blueberries earns you 10 dollars and each bushel of strawberries earns you 8 dollars. However your cart can only fit 24 bushels total and has a weight limit of 100 lbs. If a bushel of blueberries weighs 8 lbs and each bushel of strawberries weighs 6 lbs, what is your maximum profit. (You can only pick an integer number of bushels)

8. The number
$$\sqrt{2218 + 144\sqrt{35} + 176\sqrt{55} + 198\sqrt{77}}$$
can be expressed in the form $a\sqrt{5} + b\sqrt{7} + c\sqrt{11}$ for positive integers a, b, c. Find abc.

9. Let (x, y) be a point such that no circle passes through the three points $(9, 15), (12, 20), (x, y)$, and no circle passes through the points $(0, 17), (16, 19), (x, y)$. Given that $x - y = -\frac{p}{q}$ for relatively prime positive integers p, q, Find $p + q$.

10. How many ways can Alfred, Betty, Catherine, David, Emily and Fred sit around a 6 person table if no more than three consecutive people can be in alphabetical order (clockwise)?