Acton-Boxborough Math Competition Online Contest

Saturday, November 16 — Sunday, November 17, 2019

Contest Rules and Format

The 2019 November Contest consists of 10 problems — each with an answer between 0 and 100,000. The contest window is

Saturday, November 16 to Sunday, November 17.

The following rules supersede any others regarding the Online Contests.

- To submit your responses, fill out a form on our website, `abmathcompetition.org`. Include your email, name, school, grade (we require this information for prize purposes).
- Submissions close at 10:00PM Eastern Standard Time, on the final day of the round.
- Computational aids such as calculators, abaci, slide rules, etc. are prohibited. Drawing aids such as protractors and rules are permissible, but computer software such as GeoGebra or Desmos are prohibited.
- You may only work individually on the problems—consultation with others is not permitted.
- You may take as much time as you wish during the contest window.
- Each problem is worth 1 point.
- Ties will be broken by the "most difficult" problem solved. If problem A is solved by a contestants, and problem B is solved by b contestants, with $a < b$, then problem A is more difficult than B.

Awards and Prizes

- Top scoring contestants within each round will have their names posted on our website.
- Scores will be emailed to contestants within one week of the contest's end.
- Top scoring contestants across all three rounds will receive prizes.
- Physical prizes (such as AoPS gift certificates, merchandise, calculators, etc.) will be given to top scorers if they attend the ABMC onsite contest.

Good luck!

Problems

1. The remainder of a number when divided by 7 is 5. If I multiply the number by 32 and add 18 to the product, what is the new remainder when divided by 7?

2. If a fair coin is flipped 15 times, what is the probability that there are more heads than tails?

3. Let $-\frac{\sqrt{p}}{q}$ be the smallest nonzero real number such that the reciprocal of the number is equal to the number minus the square root of the square of the number, where p and q are positive integers and p is not divisible the square of any prime. Find $p+q$.

4. Rachel likes to put fertilizers on her grass to help her grass grow. However, she has cows there as well, and they eat 3 little fertilizer balls on average. If each ball is spherical with a radius of 4, then the total volume that each cow consumes can be expressed in the form $a\pi$ where a is an integer. What is a?

5. One day, all 30 students in Precalc class are bored, so they decide to play a game. Everyone enters into their calculators the expression $9 \diamondsuit 9 \diamondsuit 9 ... \diamondsuit 9$, where 9 appears 2020 times, and each $\diamondsuit$ is either a multiplication or division sign. Each student chooses the signs randomly, but they each choose one more multiplication sign than division sign. Then all 30 students calculate their expression and take the class average. Find the expected value of the class average.

6. NaNoWriMo, or National Novel Writing Month, is an event in November during which aspiring writers attempt to produce novel-length work–formally defined as 50,000 words or more–within the span of 30 days. Justin wants to participate in NaNoWriMo, but he's a busy high school student: after accounting for school, meals, showering, and other necessities, Justin only has six hours to do his homework and perhaps participate in NaNoWriMo on weekdays. On weekends, he has twelve hours on Saturday and only nine hours on Sunday, because he goes to church. Suppose Justin spends two hours on homework every single day, including the weekends. On Wednesdays, he has science team, which takes up another hour and a half of his time. On Fridays, he spends three hours in orchestra rehearsal. Assume that he spends all other time on writing. Then, if November 1st is a Friday, let w be the minimum number of words per minute that Justin must type to finish the novel. Round w to the nearest whole number.

7. Let positive reals a, b, c be the side lengths of a triangle with area 2030. Given $ab + bc + ca = 15000$ and $abc = 350000$, find the sum of the lengths of the altitudes of the triangle.

8. Find the minimum possible area of a rectangle with integer sides such that a triangle with side lengths 3,4,5, a triangle with side lengths 4,5,6, and a triangle with side lengths $\frac{9}{4}$,4,4 all fit inside the rectangle without overlapping.

9. The base 16 number $10111213...99_{16}$, which is a concatenation of all of the (base 10) 2-digit numbers, is written on the board. Then, the last $2n$ digits are erased such that the base 10 value of remaining number is divisible by 51. Find the smallest possible integer value of n.

10. Consider sequences that consist entirely of X's, Y's and Z's where runs of consecutive X's, Y's, and Z's are at most length 3. How many sequences with these properties of length 8 are there?