Team Name:________________

Round 1

1. Suppose a certain menu has 3 sandwiches and 5 drinks. How many ways are there to pick a meal so that you have exactly a drink and a sandwich?

2. If $a + b = 4$ and $a + 3b = 222222$, find $10a + b$.

3. Compute

$$\left\lfloor \frac{2019 \cdot 2017}{2018} \right\rfloor$$

where $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 2

1. Andrew has 10 water bottles, each of which can hold at most 10 cups of water. Three bottles are thirty percent filled, five are twenty-four percent filled, and the rest are empty. What is the average amount of water, in cups, contained in the ten water bottles?

2. How many positive integers divide 195 evenly?

3. Square A has side length ℓ and area 128. Square B has side length $\ell/2$. Find the length of the diagonal of Square B.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 3

1. A right triangle with area 96 is inscribed in a circle. If all the side lengths are positive integers, what is the area of the circle? Express your answer in terms of π.
2. A circular spinner has four regions labeled $3, 5, 6, 10$. The region labeled 3 is $1/3$ of the spinner, 5 is $1/6$ of the spinner, 6 is $1/10$ of the spinner, and the region labeled 10 is $2/5$ of the spinner. If the spinner is spun once randomly, what is the expected value of the number on which it lands?
3. Find the integer k such that $k^3 = 8353070389$.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 4

1. How many ways are there to arrange the letters in the word *zugzwang* such that the two z's are not consecutive?
2. If O is the circumcenter of $\triangle ABC$, $\overline{AD}$ is the altitude from A to $\overline{BC}$, $\angle CAB = 66^\circ$ and $\angle ABC = 44^\circ$, then what is the measure of $\angle OAD$?
3. If $x > 0$ satisfies $x^3 + \frac{1}{x^3} = 18$, find $x^5 + \frac{1}{x^5}$.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 5

1. Let C be the answer to Question 3. Neethen decides to run for school president! To be entered onto the ballot, however, Neethen needs $C+1$ signatures. Since no one else will support him, Neethen gets the remaining C other signatures through bribery. The situation can be modeled by $k \cdot N = 495$, where k is the number of dollars he gives each person, and N is the number of signatures he will get. How many dollars does Neethen have to bribe each person with to get exactly C signatures?
2. Let A be the answer to Question 1. With $3A-1$ total votes, Neethen still comes short in the election, losing to Serena by just 1 vote. Darn! Neethen sneaks into the ballot room, knowing that if he destroys just two ballots that voted for Serena, he will win the election. How many ways can Neethen choose two ballots to destroy?
3. Let B be the answer to Question 2. Oh no! Neethen is caught rigging the election by the principal! For his punishment, Neethen needs to run the perimeter of his school three times. The school is modeled by a square of side length k furlongs, where k is an integer. If Neethen runs B feet in total, what is $k+1$? (Note: one furlong is 1/8 of a mile).

1.________________ 2.________________ 3.________________

Team Name:________________

Round 6

1. Find the unique real positive solution to the equation $x = \sqrt{6+2\sqrt{6}+2x} - \sqrt{6-2\sqrt{6}-2x} - \sqrt{6}$.
2. Consider triangle ABC with $AB = 13$ and $AC = 14$. Point D lies on BC, and the lengths of the perpendiculars from D to AB and AC are both $\frac{56}{9}$. Find the largest possible length of BD.
3. Let $f(x,y) = \frac{m}{n}$, where m is the smallest positive integer such that x and y divide m, and n is the largest positive integer such that n divides both x and y. If $S = \{1,2,3,4,5,6,7,8,9,10\}$, what is the median of the distinct values that $f(a,b)$ can take, where $a, b \in S$?

1.________________ 2.________________ 3.________________

Team Name:________________

Round 7

1. The polynomial $y = x^4 - 22x^2 - 48x - 23$ can be written in the form

$$y = (x - \sqrt{a} - \sqrt{b} - \sqrt{c})(x - \sqrt{a} + \sqrt{b} + \sqrt{c})(x + \sqrt{a} - \sqrt{b} + \sqrt{c})(x + \sqrt{a} + \sqrt{b} - \sqrt{c})$$

for positive integers a, b, c with $a \le b \le c$. Find $(a + b) \cdot c$.

2. Varun is grounded for getting an F in every class. However, because his parents don't like him, rather than making him stay at home they toss him onto a number line at the number 3. A wall is placed at 0 and a door to freedom is placed at 10. To escape the number line, Varun must reach 10, at which point he walks through the door to freedom. Every 5 minutes a bell rings, and Varun may walk to a different number, and he may not walk to a different number except when the bell rings. Being an F student, rather than walking straight to the door to freedom, whenever the bell rings Varun just randomly chooses an adjacent integer with equal chance and walks towards it. Whenever he is at 0 he walks to 1 with a 100 percent chance. What is the expected number of times Varun will visit 0 before he escapes through the door to freedom?

3. Let $\{a_1, a_2, a_3, a_4, a_5, a_6\}$ be a set of positive integers such that every element divides 36 under the condition that $a_1 < a_2 < \cdots < a_6$. Find the probability that one of these chosen sets also satisfies the condition that every $a_i \mid a_j$ if $i|j$.

1.________________ 2.________________ 3.________________

Team Name:________________

Round 8

How many numbers between 1 and 100,000 can be expressed as the product of at most 3 distinct primes?

Your answer will be scored according to the following formula, where X is the correct answer and I is your input.

$$\max\left\{0, \left\lceil \min\left\{13 - \frac{|I - X|}{0.1|I|}, 13 - \frac{|I - X|}{0.1|I - 2X|}\right\}\right\rceil\right\}.$$

Answer:____________