Acton-Boxborough Math Competition Online Contest

Saturday, October 20 — Sunday, October 21, 2018

Contest Rules and Format

The 2018 October Contest consists of 15 problems — each with an answer between 0 and 100,000. The contest window is

Saturday, October 20 to Sunday, October 21.

The following rules supersede any others regarding the Online Contests.

- To submit your responses, fill out a form on our website, `abmathcompetition.org`. Include your email, name, school, grade (we require this information for prize purposes).
- Submissions will close at 10:00PM Eastern Standard Time, on the final day of the round.
- Computational aids such as calculators, abaci, slide rules, etc. are prohibited. Drawing aids such as protractors and rules are permissible, but computer software such as GeoGebra or Desmos are prohibited.
- You may only work individually on the problems—consultation with others is not permitted.
- You may take as much time as you wish during the contest window.
- Each problem is worth 1 point.
- Ties will be broken by the "most difficult" problem solved. If problem A is solved by a contestants, and problem B is solved by b contestants, with $a < b$, then problem A is more difficult than B.

Awards and Prizes

- Top scoring contestants within each round will have their names posted on our website.
- Scores will be emailed to contestants within one week of the contest's end.
- Top scoring contestants across all three rounds will receive prizes.
- Physical prizes (such as AoPS gift certificates, merchandise, calculators, etc.) will be given to top scorers if they attend the ABMC onsite contest.

Good luck!

Problems

1. Compute the greatest integer less than or equal to

$$\frac{10+12+14+16+18+20}{21}.$$

2. Let $A = 1, B = 2, C = 3, \ldots, Z = 26$. Find $A + B + M + C$.

3. In Mr. M's farm, there are 10 cows, 8 chickens, and 4 spiders. How many legs are there (including Mr. M's legs)?

4. The area of an equilateral triangle with perimeter 18 inches can be expressed in the form $\frac{a\sqrt{b}}{c}$, where a and c are relatively prime and b is not divisible by the square of any prime. Find $a + b + c$.

5. Let f be a linear function so $f(x) = ax + b$ for some a and b. If $f(1) = 2017$ and $f(2) = 2018$, what is $f(2019)$?

6. How many integers m satisfy $4 < m^2 \leq 216$?

7. Allen and Michael Phelps compete at the Olympics for swimming. Allen swims $\frac{9}{8}$ the distance Phelps swims, but Allen swims in $\frac{5}{9}$ of Phelps's time. If Phelps swims at a rate of 3 kilometers per hour, what is Allen's rate of swimming? The answer can be expressed as m/n for relatively prime positive integers m, n. Find $m + n$.

8. Let X be the number of distinct arrangements of the letters in "POONAM," Y be the number of distinct arrangements of the letters in "ALLEN" and Z be the number of distinct arrangements of the letters in "NITHIN." Evaluate $\frac{X+Z}{Y}$.

9. Two overlapping circles, both of radius 9cm, have centers that are 9cm apart. The combined area of the two circles can be expressed as $\frac{a\pi+b\sqrt{c}+d}{e}$ where c is not divisible by the square of any prime and the fraction is simplified. Find $a + b + c + d + e$.

10. In the Boxborough-Acton Regional High School (BARHS), 99 people take Korean, 55 people take Maori, and 27 people take Pig Latin. 4 people take both Korean and Maori, 6 people take both Korean and Pig Latin, and 5 people take both Maori and Pig Latin. 1 especially ambitious person takes all three languages, and and 100 people do not take a language. If BARHS does not offer any other languages, how many students attend BARHS?

11. Let H be a regular hexagon of side length 2. Let M be the circumcircle of H and N be the inscribed circle of H. Let m, n be the area of M and N respectively. The quantity $m - n$ is in the form πa, where a is an integer. Find a.

12. How many ordered quadruples of positive integers (p, q, r, s) are there such that $p + q + r + s \leq 12$?

13. Let $K = 2^{\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)\left(1+\frac{1}{3^8}\right)\left(1+\frac{1}{3^{16}}\right)\cdots}$. What is K^8?

14. Neetin, Neeton, Neethan, Neethine, and Neekhil are playing basketball. Neetin starts out with the ball. How many ways can they pass 5 times so that Neethan ends up with the ball?

15. In an octahedron with side lengths 3, inscribe a sphere. Then inscribe a second sphere tangent to the first sphere and to 4 faces of the octahedron. The radius of the second sphere can be expressed in the form $\frac{\sqrt{a}-\sqrt{b}}{c}$, where the square of any prime factor of c does not evenly divide into b. Compute $a + b + c$.