Team Name:________________

Round 1

1. A circle has a circumference of 20π inches. Find its area in terms of π.
2. Let x, y be the solution to the system of equations:

$$x^2 + y^2 = 10$$

$$x = 3y$$

   Find $x + y$ where both x and y are greater than zero.

3. Chris deposits \$100 in a bank account. He then spends 30% of the money in the account on biology books. The next week, he earns some money and the amount of money he has in his account increases by 30%. What percent of his original money does he now have?

1.________________ square inches 2.________________ 3.________________%

Team Name:________________

Round 2

1. The bell rings every 45 minutes. If the bell rings right before the first class and right after the last class, how many hours are there in a school day with 9 bells?
2. The middle school math team has 9 members. They want to send 2 teams to ABMC this year: one full team containing 6 members and one half team containing the other 3 members. In how many ways can they choose a 6 person team and a 3 person team?
3. Find the sum:

$$1 + (1-1)(1^2+1+1) + (2-1)(2^2+2+1) + (3-1)(3^2+3+1) + \cdots + (8-1)(8^2+8+1) + (9-1)(9^2+9+1).$$

1.________________ hours 2.________________ ways 3.________________

Team Name:________________

Round 3

1. In square $ABHI$, another square $BIEF$ is constructed with diagonal BI (of $ABHI$) as its side. What is the ratio of the area of $BIEF$ to the area of $ABHI$?
2. How many ordered pairs of positive integers (a, b) are there such that a and b are both less than 5, and the value of $ab + 1$ is prime? Recall that, for example, $(2, 3)$ and $(3, 2)$ are considered different ordered pairs.
3. Kate Lin drops her right circular ice cream cone with a height of 12 inches and a radius of 5 inches onto the ground. The cone lands on its side (along the slant height). Determine the distance between the highest point on the cone to the ground.

1.________________ 2.________________ pairs 3.________________ inches

Team Name:________________

Round 4

1. In a Museum of Fine Mathematics, four sculptures of Euler, Euclid, Fermat, and Allen, one for each statue, are nailed to the ground in a circle. Bob would like to fully paint each statue a single color such that no two adjacent statues are blue. If Bob only has only red and blue paint, in how many ways can he paint the four statues?
2. Geo has two circles, one of radius 3 inches and the other of radius 18 inches, whose centers are 25 inches apart. Let A be a point on the circle of radius 3 inches, and B be a point on the circle of radius 18 inches. If segment $\overline{AB}$ is a tangent to both circles that does not intersect the line connecting their centers, find the length of $\overline{AB}$.
3. Find the units digit to $2017^{2017!}$.

1.________________ ways 2.________________ inches 3.________________

Team Name:________________

Round 5

1. Given equilateral triangle γ_1 with vertices A, B, C, construct square $ABDE$ such that it does not overlap with γ_1 (meaning one cannot find a point in common within both of the figures). Similarly, construct square $ACFG$ that does not overlap with γ_1 and square $CBHI$ that does not overlap with γ_1. Lines DE, FG, and HI form an equilateral triangle γ_2. Find the ratio of the area of γ_2 to γ_1 as a fraction.

2. A decimal that terminates, like $1/2 = 0.5$ has a repeating block of 0. A number like $1/3 = 0.\overline{3}$ has a repeating block of length 1 since the fraction bar is only over 1 digit. Similarly, the numbers $0.0\overline{3}$ and $0.6\overline{5}$ have repeating blocks of length 1. Find the number of positive integers n less than 100 such that $1/n$ has a repeating block of length 1.

3. For how many positive integers n between 1 and 2017 is the fraction
$$\frac{n+6}{2n+6}$$
irreducible? (Irreducibility implies that the greatest common factor of the numerator and the denominator is 1.)

1.________________ 2.________________ integers 3.________________ integers

Team Name:________________

Round 6

1. Consider the binary representations of $2017, 2017 \cdot 2, 2017 \cdot 2^2, 2017 \cdot 2^3, \cdots, 2017 \cdot 2^{100}$. If we take a random digit from any of these binary representations, what is the probability that this digit is a 1?

2. Aaron is throwing balls at Carlson's face. These balls are infinitely small and hit Carlson's face at only 1 point. Carlson has a flat, circular face with a radius of 5 inches. Carlson's mouth is a circle of radius 1 inch and is concentric with his face. The probability of a ball hitting any point on Carlson's face is directly proportional to its distance from the center of Carlson's face (so when you are 2 times farther away from the center, the probability of hitting that point is 2 times as large). If Aaron throws one ball, and it is guaranteed to hit Carlson's face, what is the probability that it lands in Carlson's mouth?

3. The birth years of Atharva, his father, and his paternal grandfather form a geometric sequence. The birth years of Atharva's sister, their mother, and their grandfather (the same grandfather) form an arithmetic sequence. If Atharva's sister is 5 years younger than Atharva and all 5 people were born less than 200 years ago (from 2017), what is Atharva's mother's birth year?

1.________________ 2.________________ 3.________________

Team Name:________________

Round 7

1. A function f is called an "involution" if $f(f(x)) = x$ for all x in the domain of f and the inverse of f exists. Find the total number of involutions f with domain of integers between 1 and 8 inclusive.

2. The function $f(x) = x^3$ is an odd function since each point on $f(x)$ corresponds (through a reflection through the origin) to a point on $f(x)$. For example the point $(-2, -8)$ corresponds to $(2, 8)$.

   The function $g(x) = x^3 - 3x^2 + 6x - 10$ is a "semi-odd" function, since there is a point (a, b) on the function such that each point on $g(x)$ corresponds to a point on $g(x)$ via a reflection over (a, b). Find (a, b).

3. A permutations of the numbers $1, 2, 3, 4, 5$ is an arrangement of the numbers. For example, 12345 is one arrangement, and 32541 is another arrangement. Another way to look at permutations is to see each permutation as a function from $\{1, 2, 3, 4, 5\}$ to $\{1, 2, 3, 4, 5\}$. For example, the permutation 23154 corresponds to the function f with $f(1) = 2$, $f(2) = 3$, $f(3) = 1$, $f(5) = 4$, and $f(4) = 5$, where $f(x)$ is the xth number of the permutation.

   But the permutation 23154 has a cycle of length three since $f(1) = 2$, $f(2) = 3$, $f(3) = 1$, and cycles after 3 applications of f when regarding a set of 3 distinct numbers in the domain and range. Similarly the permutation 32541 has a cycle of length three since $f(5) = 1$, $f(1) = 3$, and $f(3) = 5$.

   In a permutation of the natural numbers between 1 and 2017 inclusive, find the expected number of cycles of length 3.

1.________________ involutions 2.________________ 3.________________ cycles

Team Name:________________

Round 8

Find the number of characters in the problems on the accuracy round test. This does not include spaces and problem numbers (or the periods after problem numbers). For example, "1. What's 5 + 10?" would contain 11 characters, namely "W," "h," "a," "t," "'," "s," "5," "+," "1," "0," "?". If the correct answer is c and your answer is x, then your score will be

$$\max\left\{0,\ 13 - \left\lceil \frac{|x-c|}{100} \right\rceil \right\}.$$

Answer:____________ characters