Name:_____ **Score:**_____/_____/**50** **Grade:** _____

The speed round is 30 minutes long. Each question is 2 points. The entire test is 50 points.

1. _____
2. _____ coins
3. _____ years
4. _____
5. _____ rhombi
6. _____ numbers
7. _____ in^2
8. _____
9. _____ students
10. _____ pairs
11. _____
12. _____
13. _____ dollars
14. _____
15. _____
16. _____
17. _____ ways
18. _____ m/s
19. _____
20. _____ dollars
21. _____
22. _____ pairs
23. _____
24. _____
25. _____

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AoPS Art of Problem Solving

MATHNASIUM

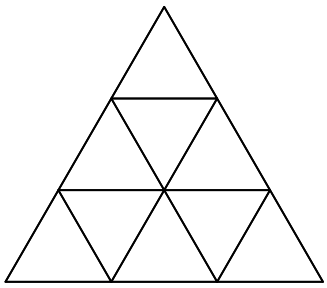
1. Find:
$$\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{4}+\frac{1}{5}}.$$

2. Rik needs to pay \$1.08 for a bottle of Mountain Dew. If he has an unlimited supply of pennies, nickels, dimes, and quarters, what is the least number of coins he can use?

3. Richard is 10 years older than his brother. Today is his 16th birthday and his brother's 6th birthday. In how many years will Richard be exactly twice as old as his brother?

4. Akshay rolls a standard six-sided die. Calculate the probability he rolls a factor of 12.

5. In the following figure of equilateral triangles, how many rhombi can Kaitlyn find?



6. Adam is writing the natural numbers on a blackboard starting with $1, 2, 3 \cdots$. When he writes the first number greater than $\sqrt{2017}$, Noah yells "stop" and Adam freezes and stops writing. How many numbers are on the board after Noah yells "stop"?

7. Consider the rectangle $ERIC$ whose length is twice its width. If the perimeter of $ERIC$ is 3 inches, find the area of the rectangle.

8. The least common multiple of some numbers is the least positive integer that is divisible by all of those numbers. What is the least common multiple of $3, 12$, and 21?

9. The Boxborough University has 100 students. 72 students study Agricultural Sciences and 31 students study Tractor Engineering. However, 2 undergraduates, Brian and Ben, do not take any classes. How many students study both Agricultural Science and Tractor Engineering?

10. Two integers are said to be "relatively prime" if the largest positive integer factor that they share is 1. Determine the number of ordered pairs (a, b), where a and b are relatively prime positive integers, such that:
$$a + b = 31.$$
(Note: $(2, 3)$ and $(3, 2)$ are considered distinct pairs.)

11. Simplify as completely as possible (write the expression without roots or powers):
$$\sqrt[n]{\frac{4^{n-3} \cdot 8^{n+2}}{3^{2n-2} \cdot 9^{n+1}}}.$$

12. What is the slope of the perpendicular bisector of a line passing through $(1, 6)$ and $(3, 4)$? Note that the perpendicular bisector of $\overline{XY}$ is the line on the coordinate plane that is equidistant to X and Y and also perpendicular to $\overline{XY}$.

13. One orange and one pineapple cost \$26. One pineapple and one apple cost \$21. One apple and one orange cost \$37. If Akshay wants to buy one orange, one pineapple, and one apple, how much money does he need?

14. If r_1 and r_2 are two distinct solutions to the quadratic $x^2 + x - 90 = 0$, find $r_1^2 + r_2^2$.

15. A right triangle has positive side lengths of $2x$, $4x - 1$, and $4x + 1$. What is the minimum possible value of x?

16. Catherine is bad at addition. Specifically, when she adds two numbers at a time, she obtains a sum 2 greater than the actual value. For example, Catherine thinks

$$1 + 2 = 5,$$

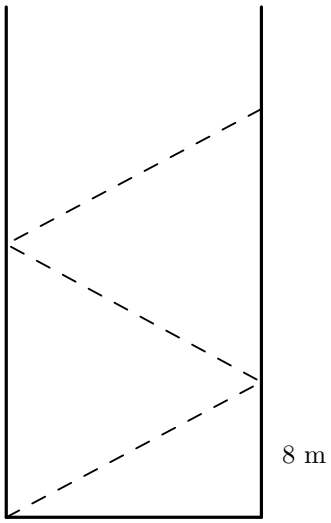
$$1 + 2 + 3 = (1 + 2) + 3 = 5 + 3 = 10,$$

$$9 + 10 = 21.$$

If Catherine computes $1 + 2 + 3 + 4 + 5 + 6$, what is the sum that she obtains?

17. Stephan is making a 12-keyed piano with 5 black keys and 7 white keys (keys of the same color are indistinguishable). How many ways can he make his piano if no two black keys are to be adjacent?

18. Iron-Man and Spider-Man are going down a straight street that is 1000 m long and 15 m wide. The street has no sidewalk and there are two long buildings right next to the street. Iron-Man flies down the street in a straight line parallel to the buildings, and Spider-Man jumps between the buildings in a zig-zag pattern as shown below (Spider-Man's path is in dotted). They both go down the entire length of the street. If Iron-Man flies at 40 m/s, and they both reach the end of the street at the same time, what is Spider-Man's speed?



19. Ol' Mr. Mutschler wakes up one day and looks out his window. He notices that his neighbors moved out and there is a family of 2 parents and 2 children (not twins) moving in. Given that he knows that at least one of the 2 children is a boy, what is the probability that the other child is a boy?

20. Antonio wants to open a pasta shop on Main Street, and he needs to decide how much to sell his pasta for. The number of customers he receives can be described by the expression $420 - 70x$, where x is the price per pound of pasta in dollars. There is another competing pasta shop on Main Street that sells pasta at \$5.50 per pound (the number of customers they get is also modeled by the same expression). If Antonio wants to get twice as many customers as the competing shop, what price should he sell his pasta at?

21. If $x + \frac{1}{x} = \sqrt{2017}$, compute the value of $x^4 + \frac{1}{x^4}$.

22. How many ordered pairs of integers (a, b) with $a, b \in [-50, 50]$ satisfy the equation:

$$a - b = \frac{(1 + a)(1 + b)}{10}.$$

23. What is the largest prime divisor of 111111111111, given that the number is divisible by a 4 digit prime?

    Note: 111111111111 has 12 ones.

24. If (x, y) lies on a circle of radius 5 with the origin as its center, find the maximum value of $2x + 3y$.

25. Two ants are on adjacent vertices of a cube. Every second, each ant randomly chooses an adjacent vertex and moves along one of the edges of the cube to that vertex. After 3 seconds, find the probability that the ants never crossed the same edge at the same time.