Acton-Boxborough Math Competition 2017 Solutions

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Special Thanks to:

Dropbox

AoPS
Art of Problem Solving

MATHNASIUM

Speed Round

Please note that we will accept answers in other forms as long as they are simplified.

1. **Problem:** Find:

$$\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{4}+\frac{1}{5}}.$$

Solution:

$$\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{4}+\frac{1}{5}} = \frac{\frac{5}{6}}{\frac{9}{20}} = \frac{5}{6}\cdot\frac{20}{9} = \frac{100}{54} = \boxed{\frac{50}{27}}.$$

Proposed by Antonio Frigo

2. **Problem:** Rik needs to pay \$1.08 for a bottle of Mountain Dew. If he has an unlimited supply of pennies, nickels, dimes, and quarters, what is the least number of coins he can use?

Solution: Since we want to minimize the total number of coins used, we always try to use the most valuable coin first. The optimal arrangement is 4 quarters, 0 dimes, 1 nickel, and 3 pennies, making the minimum number of coins $\boxed{8}$.

Proposed by Akshay Gowrishankar

3. **Problem:** Richard is 10 years older than his brother. Today is his 16th birthday and his brother's 6th birthday. In how many years will Richard be exactly twice as old as his brother?

Solution: In x years, $6+x$ is Richard's brother's age and $16+x$ is Richard's age. Our equation is:

$$2(6+x) = 16+x.$$

Solving this yields the answer of $x = \boxed{4}$ years.

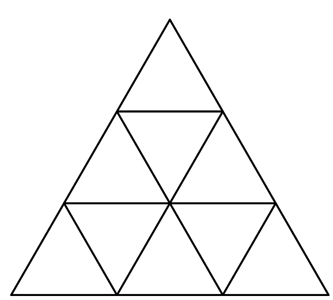
Proposed by Richard Huang

4. **Problem:** Akshay rolls a standard six-sided die. Calculate the probability he rolls a factor of 12.

Solution: The factors of 12 on a die are 1, 2, 3, 4, and 6, so there are 5 potential factors the die can roll. Since there are 6 total possibilities on a standard die, the probability is $\boxed{5/6} = \boxed{0.8\overline{3}}$.

Proposed by David Lu

5. **Problem:** In the following figure of equilateral triangles, how many rhombi can Kaitlyn find?



Solution: Clearly, the maximum size of each rhombus is having a side length equal to that of one edge of an equilateral triangle. There are 3 rhombi with sides parallel to the left and right sides of the grid, 3 with sides parallel to the bottom and right, and 3 with sides parallel to the bottom and left. We find that the answer is $\boxed{9}$ rhombi.

Proposed by David Lu and Allen Wang

6. **Problem:** Adam is writing the natural numbers on a blackboard starting with $1, 2, 3 \cdots$. When he writes the first number greater than $\sqrt{2017}$, Noah yells "stop" and Adam freezes and stops writing. How many numbers are on the board after Noah yells "stop"?

   Solution: Adam stopped at the first integer greater than $\sqrt{2017}$. The square of 45 is 2025 and the square of 44 is 1936, so the smallest integer greater than $\sqrt{2017}$ is 45. At this point, there are $\boxed{45}$ numbers on the board.

   Proposed by Allen Wang

7. **Problem:** Consider the rectangle $ERIC$ whose length is twice its width. If the perimeter of $ERIC$ is 3 inches, find the area of the rectangle.

   Solution: Let w be the width of the rectangle and ℓ be its length. Therefore:

$$3 = 2(w + \ell) = 2(w + 2w) \implies w = \frac{1}{2}.$$

   So, the length is 1 inch. The area is then $1 \cdot \frac{1}{2} = \boxed{\frac{1}{2}} = \boxed{0.5}$ square inches.

   Proposed by David Lu

8. **Problem:** The least common multiple of some numbers is the least positive integer that is divisible by all of those numbers. What is the least common multiple of $3, 12,$ and 21?

   Solution: Note that both 12 and 21 contain factors of 3 in them, and $12 = 3 \cdot 4$ while $21 = 3 \cdot 7$. Therefore, the LCM of the three numbers is $3 \cdot 4 \cdot 7 = \boxed{84}$.

   Proposed by David Lu

9. **Problem:** The Boxborough University has 100 students. 72 students study Agricultural Sciences and 31 students study Tractor Engineering. However, 2 undergraduates, Brian and Ben, do not take any classes. How many students study both Agricultural Science and Tractor Engineering?

   Solution: We apply the Principle of Inclusion-Exclusion. The number of students who are majoring in both subjects is equal to the total number of students taking at least 1 class minus the sum of the students taking Agricultural Science and the students taking Tractor Engineering:

$$98 = 72 + 31 - n$$
$$n = \boxed{5} \text{ students.}$$

   Proposed by Akshay Gowrishankar

10. **Problem:** Two integers are said to be "relatively prime" if the largest positive integer factor that they share is 1. Determine the number of ordered pairs (a, b), where a and b are relatively prime positive integers, such that:

$$a + b = 31.$$

(Note: $(2, 3)$ and $(3, 2)$ are considered distinct pairs.)

Solution: Let's just see what happens if a and b did share a common factor $k > 0$. Then $a = a'k$ and $b = b'k$ for some positive integers a' and b', so the equation would become:

$$a + b = a'k + b'k = k(a' + b') = 31.$$

But note that this implies that k divides 31 $\implies k = 1$ or 31 since 31 is prime. Because the latter is impossible, we know that $k = 1$. But this means that no matter what combination of a and b we choose such that $a + b = 31$, the greatest common factor of a and b must be 1. In other words, a and b are relatively prime.

Therefore, all ordered pairs of positive integers (a, b) work as long as $a + b = 31$. The ordered pairs are $(1, 30), (2, 29), \cdots, (30, 1)$ so there are $\boxed{30}$ of them.

Proposed by Antonio Frigo

11. **Problem:** Simplify as completely as possible (write the expression without roots or powers):

$$\sqrt[n]{\frac{4^{n-3} \cdot 8^{n+2}}{3^{2n-2} \cdot 9^{n+1}}}.$$

Solution: We have:

$$\sqrt[n]{\frac{2^{2n-6} \cdot 2^{3n+6}}{3^{2n-2} \cdot 3^{2n+2}}} = \sqrt[n]{\frac{2^{5n}}{3^{4n}}} = \frac{2^5}{3^4} = \boxed{\frac{32}{81}}.$$

Proposed by Antonio Frigo

12. **Problem:** What is the slope of the perpendicular bisector of a line passing through $(1, 6)$ and $(3, 4)$? Note that the perpendicular bisector of $\overline{XY}$ is the line on the coordinate plane that is equidistant to X and Y and also perpendicular to $\overline{XY}$.

Solution: The slope of the perpendicular bisector is merely the opposite reciprocal of the line containing the two points. The slope of the original line is:

$$\frac{4 - 6}{3 - 1} = -1.$$

The opposite reciprocal of this is $\boxed{1}$.

Proposed by Antonio Frigo and Akshay Gowrishankar

13. **Problem:** One orange and one pineapple cost \$26. One pineapple and one apple cost \$21. One apple and one orange cost \$37. If Akshay wants to buy one orange, one pineapple, and one apple, how much money does he need?

Solution: Let p be the cost of a pineapple, σ be the cost of an orange, and a be the cost of an apple. We have:

$$p + a = 21$$

$$a + \sigma = 37$$

$$\sigma + p = 26.$$

Adding these all together:

$$2(p + a + \sigma) = 84.$$

$$p + a + \sigma = 42$$

Thus, the cost of one apple, pineapple, and orange is $\boxed{42}$ dollars.

Proposed by Akshay Gowrishankar

14. **Problem:** If r_1 and r_2 are two distinct solutions to the quadratic $x^2+x-90=0$, find $r_1^2+r_2^2$.

    Solution: We factor the quadratic to get

    $$(x-9)(x+10) = 0,$$

    so r_1 and r_2 are the roots of the quadratic, 9 and -10. Then

    $$r_1^2 + r_2^2 = 9^2 + (-10)^2 = \boxed{181}.$$

    Alternatively, since r_1 and r_2 are solutions, they satisfy the equation to the quadratic, so $r_1^2 + r_1 - 90 = 0$ and $r_2^2 + r_2 - 90 = 0$. Adding these two equations and rearranging gives:

    $$r_1^2 + r_2^2 = 180 - (r_1 + r_2).$$

    Also, by the fundamental theorem of algebra, the quadratic can be factored with its roots. We can write $x^2 + x - 90 = (x - r_1)(x - r_2) = x^2 - (r_1 + r_2)x + r_1 r_2$; here we can match up coefficients to find the sum of the roots. Therefore:

    $$r_1^2 + r_2^2 = 180 - (r_1 + r_2) = 180 - (-1) = 181.$$

    Proposed by Antonio Frigo

15. **Problem:** A right triangle has positive side lengths of $2x$, $4x-1$, and $4x+1$. What is the minimum possible value of x?

    Solution: Clearly, $4x+1 > 4x-1$ and $4x+1 > 2x$ whenever $x > 0$. But since x is positive, we find that $4x+1$, the longest length, must be the hypotenuse. By the Pythagorean Theorem:

    $$(2x)^2 + (4x-1)^2 = (4x+1)^2$$

    $$4x^2 + 16x^2 - 8x + 1 = 16x^2 + 8x + 1$$

    $$4x^2 - 16x = 0$$

    $$4x(x-4) = 0$$

    $$x = 4, 0$$

    Plugging in $x = 0$ gives negative side lengths, but $x = \boxed{4}$ produces an 8 by 15 by 17 triangle.

    Proposed by Akshay Gowrishankar

16. **Problem:** Catherine is bad at addition. Specifically, when she adds two numbers at a time, she obtains a sum 2 greater than the actual value. For example, Catherine thinks

$$1 + 2 = 5,$$

$$1 + 2 + 3 = (1 + 2) + 3 = 5 + 3 = 10,$$

$$9 + 10 = 21.$$

If Catherine computes $1 + 2 + 3 + 4 + 5 + 6$, what is the sum that she obtains?

Solution: We have:

$$((((1+2)+3)+4)+5)+6 = (((5+3)+4)+5)+6 = ((10+4)+5)+6 = (16+5)+6 = 23+6 = \boxed{31}.$$

But a more meaningful solution (to save time on the speed round) is to notice that every time Catherine encounters a "+" sign she adds 2 more. In the original sum $1 + 2 + 3 + 4 + 5 + 6$, she is supposed to obtain 21. Since there are 5 addition symbols, she obtains the sum $2 \cdot 5$ more or $21 + 10 = \boxed{31}$.

Proposed by Allen Wang

17. **Problem:** Stephan is making a 12-keyed piano with 5 black keys and 7 white keys (keys of the same color are indistinguishable). How many ways can he make his piano if no two black keys are to be adjacent?

Solution: We can put the 5 black keys down with 1 white key between each of the two black keys to produce the sequence $BWBWBWBWB$. We see that there are 6 locations for the remaining 3 white keys. We distribute these white keys to these 6 locations in $\binom{8}{3} = \boxed{56}$ ways. So Stephan has a total of 56 ways to make his piano.

Proposed by Allen Wang

18. **Problem:** Iron-Man and Spider-Man are going down a straight street that is 1000 m long and 15 m wide. The street has no sidewalk and there are two long buildings right next to the street. Iron-Man flies down the street in a straight line parallel to the buildings, and Spider-Man jumps between the buildings in a zig-zag pattern as shown below (Spider-Man's path is in dotted). They both go down the entire length of the street. If Iron-Man flies at 40 m/s, and they both reach the end of the street at the same time, what is Spider-Man's speed?

Solution: Let us break up the velocity of Spider-Man into its components. Using the Pythagorean Theorem, we find that the diagonal length is 17. Therefore, for every 17 m Spider-Man travels, he only travels 8 m forward, so he must travel $\frac{17}{8}$ times faster than Iron-Man to go the same distance forward. Thus, Spider-Man's speed is $40 \cdot \frac{17}{8} = \boxed{85}$ m/s.

Proposed by Akshay Gowrishankar

19. **Problem:** Ol' Mr. Mutschler wakes up one day and looks out his window. He notices that his neighbors moved out and there is a family of 2 parents and 2 children (not twins) moving in. Given that he knows that at least one of the 2 children is a boy, what is the probability that the other child is a boy?

Solution: Let B represent a boy and G represent a girl. Originally, there are 4 equally weighted options: BB, BG, GB, or GG, where the first letter in each pair of letters is the younger child. Note that BG and GB are different since we do not know which child is older. Since we are given that at least one of the children is a boy, the GG option is impossible, leaving 3 equally weighted options, one of which is BB. So the probability is $\boxed{\frac{1}{3}}$.

Proposed by Allen Wang

20. **Problem:** Antonio wants to open a pasta shop on Main Street, and he needs to decide how much to sell his pasta for. The number of customers he receives can be described by the expression $420 - 70x$, where x is the price per pound of pasta in dollars. There is another competing pasta shop on Main Street that sells pasta at \$5.50 per pound (the number of customers they get is also modeled by the same expression). If Antonio wants to get twice as many customers as the competing shop, what price should he sell his pasta at?

    Solution: The competing pasta shop receives $420 - 70 \cdot 5.50 = 420 - 385 = 35$ customers. Antonio wants to get $35 \cdot 2 = 70$ customers. Therefore, we have

    $$420 - 70x = 70 \implies 70x = 350 \implies x = 5.$$

    Antonio should sell his pasta for $\boxed{5}$ dollars.

    Proposed by Akshay Gowrishankar

21. **Problem:** If $x + \frac{1}{x} = \sqrt{2017}$, compute the value of $x^4 + \frac{1}{x^4}$.

    Solution: This problem tests students' computational skills as well as algebraic manipulation skills. We see that

    $$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 2017 - 2 = 2015.$$

    Similarly,

    $$x^4 + \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 2015^2 - 2 = \boxed{4060223}.$$

    Proposed by Akshay Gowrishankar

22. **Problem:** How many ordered pairs of integers (a, b) with $a, b \in [-50, 50]$ satisfy the equation:

    $$a - b = \frac{(1+a)(1+b)}{10}.$$

    Solution: We begin manipulating the equation

    $$\begin{aligned} a - b &= \frac{(1+a)(1+b)}{10} \\ \implies 10a - 10b &= 1 + a + b + ab \\ \implies ab - 9a + 11b + 1 &= 0 \end{aligned}$$

Using a neat factoring technique (known often as Simon's Favorite Factoring Trick), we rewrite the equation as

$$(a + 11)(b - 9) + 100 = 0 \implies (a + 11)(b - 9) = -100.$$

So $a + 11$ and $b - 9$ must both be integers that multiply to -100.

We go through all of the integers that multiply to -100 and remove the ones in which $a + 11$ and $b - 9$ lie outside of the given interval.

We find that

$$(a + 11, b - 9) \in \{(-25, 4), (-20, 5), (-10, 10), (-5, 20), (-4, 25), (2, -50), (4, -25),$$
$$(5, -20), (10, -10), (20, -5), (25, -4), (50, -2)\}.$$

Thus, we have $\boxed{12}$ pairs.

Proposed by Antonio Frigo and Richard Huang

23. **Problem:** What is the largest prime divisor of 111111111111, given that the number is divisible by a 4 digit prime?

    Note: 111111111111 has 12 ones.

    Solution: We see that 9 times 111111111111 gives 999999999999, which equals $10^{12} - 1$. So the sequence of 12 ones is therefore equal to

    $$\frac{10^{12} - 1}{9}.$$

    This we can factor as a difference of squares then another difference of squares and a sum of cubes:

    $$\frac{10^{12} - 1}{9} = \frac{(10^6 - 1)(10^6 + 1)}{9} = \frac{(10^3 - 1)(10^3 + 1)(10^2 + 1)(10^4 - 10^2 + 1)}{9}.$$

    We take a closer look at the numerator. $10^3 - 1 = 999 = 9 \cdot 111 = 9 \cdot 3 \cdot 37$. For $10^3 + 1$, we know that $10 + 1 = 11$ must divide it so $1001 = 11 \cdot 7 \cdot 13$. $10^2 + 1 = 101$, which is prime and $10^4 - 10^2 + 1 = 9901$. This seems to be the largest factor yet. Since all other prime factors are less than 4 digits, we know that $\boxed{9901}$ is the 4 digit prime factor.

    Proposed by Allen Wang

24. **Problem:** If (x, y) lies on a circle of radius 5 with the origin as its center, find the maximum value of $2x + 3y$.

    Solution: Let (x, y) lie on the line $2x + 3y = C$. Our goal is to maximize C. Note that (x, y) must satisfy the equation of this line and the equation of the circle. To maximize C, it is optimal for the line $2x + 3y = C$ to be tangent to the circle of radius 5 centered at the origin.

    Since the line connecting the point of tangency to the center of the circle is perpendicular to the line $2x + 3y = C$, the segment from that point of tangency to the center should have a slope of $\frac{3}{2}$. Thus, the equation of the line connecting the center of the circle and the point of tangency is

$$y = \frac{3}{2}x.$$

The equation of the circle is

$$x^2 + y^2 = 25.$$

Solving gives that the point of tangency of the circle is either $(\frac{10}{\sqrt{13}}, \frac{15}{\sqrt{13}})$ or $(-\frac{10}{\sqrt{13}}, -\frac{15}{\sqrt{13}})$. Clearly, the positive coordinates yield a larger C, so the line passes through $(\frac{10}{\sqrt{13}}, \frac{15}{\sqrt{13}})$. We find that

$$2x + 3y = \frac{20}{\sqrt{13}} + \frac{45}{\sqrt{13}} = \frac{65}{\sqrt{13}} = \boxed{5\sqrt{13}}.$$

Proposed by Antonio Frigo

25. **Problem:** Two ants are on adjacent vertices of a cube. Every second, each ant randomly chooses an adjacent vertex and moves along one of the edges of the cube to that vertex. After 3 seconds, find the probability that the ants never crossed the same edge at the same time.

Solution: First, observe that the ants can never be on the same vertex. Moreover, the ants can either be on adjacent vertices or opposite vertices, where opposite means that they are $s\sqrt{3}$ units apart where s is the side length of the cube.

If the ants are in an adjacent position, we can see that after 1 second there is a

- $\frac{1}{9}$ probability that they cross
- $\frac{2}{9}$ probability that they end up on opposite vertices
- $\frac{2}{3}$ probability that they end up on adjacent vertices (and not cross)

If the ants are in an opposite position, we see that after 1 second there is a

- $\frac{1}{3}$ probability they end up on opposite vertices
- $\frac{2}{3}$ probability they end up on adjacent vertices (and not cross)
- 0 probability that they cross

During the 3 seconds, the two ants each visit three (not necessarily distinct) vertices. If they cross, they must cross on the first, second, or third move. If the ants cross, we see that the sequence of moves for the ants during those 3 seconds are (with O meaning the ants are on opposite vertices and A meaning they are on adjacent vertices):

- Cross → Any position → Any position
- A → Cross → Any position
- O → A → Cross
- A → A → Cross

The probability of the ants crossing is the sum of these four cases, which is

$$\frac{1}{9} \cdot 1 \cdot 1 + \frac{2}{3} \cdot \frac{1}{9} \cdot 1 + \frac{2}{9} \cdot \frac{2}{3} \cdot \frac{1}{9} + \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{9} = \frac{61}{243}.$$

But this is the probability that the ants do cross. So the complement, $1 - \frac{61}{243} = \boxed{\dfrac{182}{243}}$, is the answer.

Proposed by Allen Wang

Accuracy Round

Please note that we will accept answers in other forms as long as they are simplified.

1. **Problem:** Len's Spanish class has four tests in the first term. Len scores 72, 81, and 78 on the first three tests. If Len wants to have an 80 average for the term, what is the minimum score he needs on the last test?

   Solution: If Len's wants an 80 average across four tests, then he must score a minimum of $80 \cdot 4 = 320$ total points.

   Let the minimum score Len needs on his fourth test be m. Then

   $$72 + 81 + 78 + m = 320,$$

   so $m = 320 - 72 - 81 - 78 = \boxed{89}$ points.

   Proposed by Akshay Gowrishankar

2. **Problem:** In 1824, the Electoral College had 261 members. Andrew Jackson won 99 Electoral College votes and John Quincy Adams won 84 votes. A plurality occurs when no candidate has more than 50% of the votes. Should a plurality occur, the vote goes to the House of Representatives to break the tie. How many more votes would Jackson have needed so that a plurality would not have occurred?

   Solution: To have at least 50% of 261 total votes, Jackson would have needed a minimum 131 votes. Since Jackson received 99 votes, he would have needed $131 - 99 = \boxed{32}$ more votes.

   Proposed by Antonio Frigo

3. **Problem:** $1/2 + 1/6 + 1/12 + 1/20 + 1/30 = 1 - 1/n$. Find n.

   Solution: We have $1/2 = 1 - 1/2$, $1/6 = 1/2 - 1/3$, $1/12 = 1/3 - 1/4$, $1/20 = 1/4 - 1/5$, and $1/30 = 1/5 - 1/6$. Therefore, the sum equals

   $$1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \frac{1}{5} - \frac{1}{6} = 1 - \frac{1}{6}.$$

   In particular, we find the the value of n is $\boxed{6}$.

   Proposed by Allen Wang

4. **Problem:** How many ways are there to sit Samuel, Esun, Johnny, and Prat in a row of 4 chairs if Prat and Johnny refuse to sit on an end?

   Solution: Since Prat and Johnny refuse to sit on an end, they must occupy the middle 2 seats. It follows that Samuel and Esun must occupy the 2 end seats.

   There are 2 ways for Samuel and Esun to sit. Either Samuel can occupy the leftmost seat and Esun can occupy the rightmost seat, or Samuel can occupy the rightmost seat and Esun can occupy the leftmost seat. Similarly, there are 2 ways for Prat and Johnny to sit.

   Therefore, there are $2 \cdot 2 = \boxed{4}$ total arrangements.

   Proposed by David Lu

5. **Problem:** Find an ordered quadruple (w, x, y, z) that satisfies the following:

$$3^w + 3^x + 3^y = 3^z$$

where $w + x + y + z = 2017$.

Solution: Observe that

$$3^a + 3^a + 3^a = 3 \cdot 3^a = 3^{a+1}$$

for any a. Thus, the equation $3^w + 3^x + 3^y = 3^z$ is satisfied when $(w, x, y, z) = (a, a, a, a+1)$.

We can therefore let $w = a, x = a, y = a$, and $z = a+1$. Substituting into $w+x+y+z = 2017$ gives

$$a + a + a + (a + 1) = 2017.$$

Solving yields $a = 504$, so

$$(w, x, y, z) = (a, a, a, a + 1) = \boxed{(504, 504, 504, 505)}.$$

Proposed by Antonio Frigo

6. **Problem:** In rectangle $ABCD$, E is the midpoint of CD. If $AB = 6$ inches and $AE = 6$ inches, what is the length of AC?

Solution: Draw a picture. $AB = 6$ and $ABCD$ is a rectangle, so $CD = 6$. E is the midpoint of CD, so $DE = 3$.

By the Pythagorean Theorem, $AD = \sqrt{AE^2 - DE^2} = \sqrt{6^2 - 3^2} = 3\sqrt{3}$. Using the Pythagorean Theorem again, we get $AC = \sqrt{AD^2 + CD^2} = \sqrt{(3\sqrt{3})^2 + 6^2} = \sqrt{63} = \boxed{3\sqrt{7}}$.

Proposed by Akshay Gowrishankar

7. **Problem:** Call an integer interesting if the integer is divisible by the sum of its digits. For example, 27 is divisible by $2 + 7 = 9$, so 27 is interesting. How many 2-digit interesting integers are there?

Solution: We start from 10 and count up, since there really aren't that many 2-digit numbers. We note that $10, 12, 18, 20, 21, 24, 27, 30, 36, 40, 42, 45, 48, 50, 54, 60, 63, 70, 72, 80, 81, 84, 90$ all work, providing $\boxed{23}$ solutions.

Proposed by Chris Wang

8. **Problem:** Let $a\#b = \frac{a^3-b^3}{a-b}$. If a, b, c are the roots of the polynomial $x^3 + 2x^2 + 3x + 4$, what is the value of $a\#b + b\#c + c\#a$?

Solution: Note that $a\#b = \frac{a^3-b^3}{a-b} = a^2 + ab + b^2$, so we have $a\#b + b\#c + c\#a = 2a^2 + 2b^2 + 2c^2 + ab + bc + ca$. We can rewrite this as

$$2(a + b + c)^2 - 3(ab + bc + ca).$$

By Vieta's Formulas, we know

$$a + b + c = -2$$

and

$$ab + bc + ca = 3.$$

Therefore,

$$\begin{aligned} a\#b + b\#c + c\#a &= 2a^2 + 2b^2 + 2c^2 + ab + bc + ca \\ &= 2(a+b+c)^2 - 3(ab+bc+ca) \\ &= 2 \cdot (-2)^2 - 3 \cdot 3 \\ &= 8 - 9 = \boxed{-1}. \end{aligned}$$

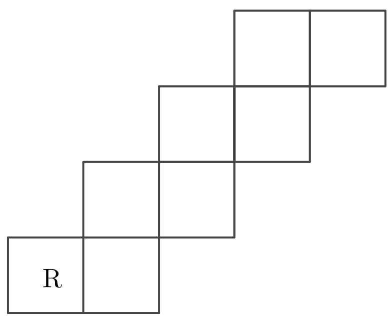
Afternote: If r_1, r_2, r_3 are the roots of the equation, then we know that $(x-r_1)(x-r_2)(x-r_3) = x^3 + 2x^2 + 3x + 4$. But we can expand the left side of the equation to get $x^3 - (r_1 + r_2 + r_3)x^2 + (r_1r_2 + r_2r_3 + r_3r_1)x - r_1r_2r_3$ so we can match up our coefficients to obtain the values for the sum and products of the roots, proving Vieta's Formulas.

Proposed by Akshay Gowrishankar

9. **Problem:** Akshay and Gowri are examining a strange chessboard. Suppose 3 distinct rooks are placed into the following chessboard. Find the number of ways that one can place these rooks so that they don't attack each other. Note that two rooks are considered attacking each other if they are in the same row or the same column.

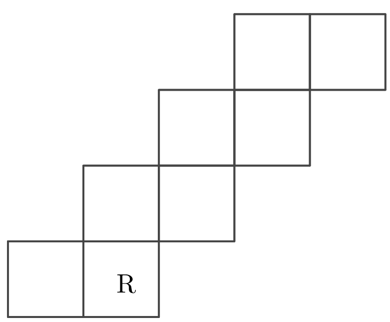
   Solution: We consider 4 cases for where the lowest rook is placed. Call the lowest rook R.

   Case 1:



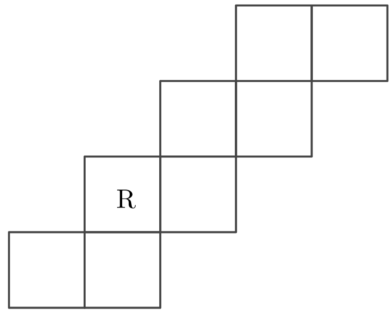
   The other 2 rooks can be placed anywhere except on R or in the adjacent space. There are $\binom{6}{2} = 15$ ways to place the rooks, but there are 5 ways to place them and make them adjacent, giving us $15 - 5 = 10$ ways to place the rooks.

   Case 2:



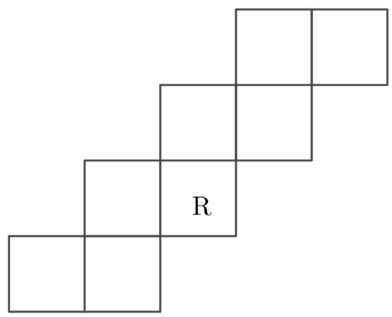
   Similarly, there are $\binom{5}{2} = 10$ ways to choose where the rooks are placed, but 4 ways for them to be adjacent, so $10 - 4 = 6$ ways exist.

   Case 3:



Similarly, there are $\binom{4}{2} = 6$ ways to choose where the rooks are placed, but 3 ways for them to be adjacent, so $6 - 3 = 3$ ways exist.

Case 4:



Similarly, there are $\binom{3}{2} = 3$ ways to choose where the rooks are placed, but 2 ways for them to be adjacent, so $3 - 2 = 1$ ways exist

In total, we have $10 + 6 + 3 + 1 = 20$ ways to place 3 rooks into the following chessboard so they do not attack each other. But note that the problem says that the rooks are distinct, so we must multiply by a factor of 3! to obtain the final answer of $\boxed{120}$.

Proposed by Allen Wang and Richard Huang

10. **Problem:** The Earth is a very large sphere. Richard and Allen have a large spherical model of Earth, and they would like to (for some strange reason) cut the sphere up with planar cuts. If each cut intersects the sphere, and Allen holds the sphere together so it does not fall apart after each cut, what is the maximum number of pieces the sphere can be cut into after 6 cuts?

    Solution: The problem is equivalent to asking the maximum number of regions produced by 6 planes in sphere. We arrange these planes in general position: the position where all pairs of planes intersect in a line, and all triples intersect at a point. Clearly, this arrangement of planes produces the most regions.

    Now we count the 3-dimensional regions. Imagine the point $(1, 1, 1)$ in $\mathbb{R}^3$. Let 3 planes be $x = 0$, $y = 0$, and $z = 0$. These planes are in general position. Also, it is clear that there are 8 regions. Yet, there is a nicer way to count these regions. Remember that point at $(1, 1, 1)$? Consider the sphere of radius 0 centered at $(1, 1, 1)$. Now consider blowing into that sphere so its radius increases. As that sphere touches the plane $x = 0$ or $y = 0$ or $z = 0$, it passes into a new region. But when the sphere touches a line of intersection, it passes into another region, and when it becomes tangent to the origin, it passes into another region.

From this we see that there is a 1-to-1 correspondence between the number of 3-dimensional regions we are adding and the sum of the number of planes, lines of intersection, and the points of intersection. We can apply the same logic here: for 6 planes, we have obviously 6 planes, $\binom{6}{2}$ lines of intersection, and $\binom{6}{3}$ points of intersection. Adding these to the original region that our chosen point was in shows that the total number of pieces that the sphere can be cut into is

$$\binom{6}{0} + \binom{6}{1} + \binom{6}{2} + \binom{6}{3} = \boxed{42}.$$

In addition one must note that M is the 13th letter of the alphabet, A is the first, T is the 20th, and H is the 8th. Moreover, $13 + 1 + 20 + 8 = 42$, the meaning to life and everything.

Proposed by Allen Wang

Team Round

Please note that we will accept answers in other forms as long as they are simplified.

Round 1

1. **Problem:** A circle has a circumference of 20π inches. Find its area in terms of π.

   Solution: Since the circumference $d\pi = 20\pi$, the diameter is 20, which gives a radius of 10. So the area is $10^2\pi = \boxed{100\pi}$ square inches.

   Proposed by Akshay Gowrishankar

2. **Problem:** Let x, y be the solution to the system of equations:

   $$x^2 + y^2 = 10$$

   $$x = 3y$$

   Find $x + y$ where both x and y are greater than zero.

   Solution: We substitute $x = 3y$ into the first equation to get $(3y)^2 + y^2 = 10 \implies 10y^2 = 10$. So $y = 1$ and $x = 3$, implying that $x + y = \boxed{4}$.

   Proposed by Richard Huang

3. **Problem:** Chris deposits \$100 in a bank account. He then spends 30% of the money in the account on biology books. The next week, he earns some money and the amount of money he has in his account increases by 30%. What percent of his original money does he now have?

   Solution: He begin with 100 dollars. After he spends 30% he has 70 dollars remaining. He gains a 30% so he now has

   $$70 \cdot \frac{13}{10} = 91$$

   dollars, which is $\boxed{91}$% of what he began with.

   Proposed by Akshay Gowrishankar

Round 2

1. **Problem:** The bell rings every 45 minutes. If the bell rings right before the first class and right after the last class, how many hours are there in a school day with 9 bells?

   Solution: Each bell except for the last bell marks the beginning of another 45 minute class. Since there are 9 bells, there are 8 classes, so the school day is

   $$8 \cdot \frac{45}{60} = \boxed{6} \text{ hours long.}$$

   Proposed by Allen Wang

2. **Problem:** The middle school math team has 9 members. They want to send 2 teams to ABMC this year: one full team containing 6 members and one half team containing the other 3 members. In how many ways can they choose a 6 person team and a 3 person team?

   Solution: Notice that after we choose the 3 person team, the other 6 members must be the other team, so the question simply requires us to choose 3 of the 9 people. And

$$\binom{9}{3} = \boxed{84}.$$

   Proposed by Akshay Gowrishankar

3. **Problem:** Find the sum:

$$1+(1-1)(1^2+1+1)+(2-1)(2^2+2+1)+(3-1)(3^2+3+1)+\cdots+(8-1)(8^2+8+1)+(9-1)(9^2+9+1).$$

   Solution: First, we can expand $(n-1)(n^2+n+1) = n^3 - 1$. So the sum equals:

$$1 + 1^3 - 1 + 2^3 - 1 + \cdots 9^3 - 1 = (1^3 + 2^3 + \cdots + 9^3) - 8.$$

   But the sum of the first 9 cubes is $45^2 = 2025$, which can be computed directly or via the formula. Subtracting 8 gives the answer of $\boxed{2017}$.

   Proposed by Allen Wang

Round 3

1. **Problem:** In square $ABHI$, another square $BIEF$ is constructed with diagonal BI (of $ABHI$) as its side. What is the ratio of the area of $BIEF$ to the area of $ABHI$?

   Solution: The ratio of the side lengths of the squares is $\sqrt{2} : 1$ by the Pythagorean Theorem so the ratio of their areas is this ratio squared, or $\boxed{2{:}1}$. 2 and $\frac{2}{1}$ are also acceptable.

   Proposed by David Lu

2. **Problem:** How many ordered pairs of positive integers (a, b) are there such that a and b are both less than 5, and the value of $ab + 1$ is prime? Recall that, for example, $(2, 3)$ and $(3, 2)$ are considered different ordered pairs.

   Solution: The most straightforward way to solve this problem is to try out all possibilities. But note that a and b, when not both equal to 1 cannot both be odd numbers, this reduces our search by a bit.

   The ordered pairs are: $(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (2, 3), (3, 2), (3, 4), (4, 1), (4, 3), (4, 4)$. There are $\boxed{11}$ of them.

   Proposed by Allen Wang

3. **Problem:**

   Kate Lin drops her right circular ice cream cone with a height of 12 inches and a radius of 5 inches onto the ground. The cone lands on its side (along the slant height). Determine the distance between the highest point on the cone to the ground.

Solution: When the cone falls on its side, we consider the cross section of the cone that is a triangle that contains the vertex of the cone, the tangency line of the cone with the ground, and the highest point on the cone (this point is at its base).

This cross section is an isosceles $13 \times 13 \times 10$ triangle, the problem asks for its altitude h to the side of length 13. The area of the triangle is $10 \cdot 12/2 = 60$.

We have

$$\frac{bh}{2} = A \implies h = \frac{2A}{b} = \frac{2 \cdot 60}{13} = \boxed{\frac{120}{13}} \text{ inches.}$$

Proposed by Allen Wang

Round 4

1. **Problem:** In a Museum of Fine Mathematics, four sculptures of Euler, Euclid, Fermat, and Allen, one for each statue, are nailed to the ground in a circle. Bob would like to fully paint each statue a single color such that no two adjacent statues are blue. If Bob only has only red and blue paint, in how many ways can he paint the four statues?

   Solution: There are only a couple of cases. When 4 statues are red, he has only 1 possible painting. When 3 are red, there are 4 possible ways to paint. When 2 are red, there are 2 ways to paint. This gives a total of $\boxed{7}$ ways to paint.

   Proposed by Allen Wang

2. **Problem:** Geo has two circles, one of radius 3 inches and the other of radius 18 inches, whose centers are 25 inches apart. Let A be a point on the circle of radius 3 inches, and B be a point on the circle of radius 18 inches. If segment $\overline{AB}$ is a tangent to both circles that does not intersect the line connecting their centers, find the length of $\overline{AB}$.

   Solution: Let X be the center of the circle of radius 3 and let Y be the center of the circle of radius 18. Since the tangent and the radius are perpendicular, we find that XA and YB are both perpendicular to AB, so they are parallel. Consider the foot of the altitude from X to YB, let that point be Z.

   From the parallelogram $XZBA$ we know that XZ and BA are equal. Also XZY is a right triangle, specifically, a $15-20-25$ triangle, so XZ has a length of 20, and $AB = \boxed{20}$ inches.

   Proposed by Antonio Frigo

3. **Problem:** Find the units digit to $2017^{2017!}$.

   Solution: Since we are only finding the units digit, we can compute the units digit of $7^{2017!}$. Let $2017!/4 = k$, an integer; then we see that:

   $$7^{2017!} = 7^{4 \cdot k} = (7^4)^k = 2401^k.$$

   But the powers of 2401 always end in a $\boxed{1}$, which is the units digit.

   Proposed by Allen Wang

Round 5

1. **Problem:** Given equilateral triangle γ_1 with vertices A, B, C, construct square $ABDE$ such that it does not overlap with γ_1 (meaning one cannot find a point in common within both of the figures). Similarly, construct square $ACFG$ that does not overlap with γ_1 and square $CBHI$ that does not overlap with γ_1. Lines DE, FG, and HI form an equilateral triangle γ_2. Find the ratio of the area of γ_2 to γ_1 as a fraction.

   Solution: Do not fear, Greek letters are here!

   We simply need to find the ratio of the side lengths and then square the ratio. Let the center of γ_1 and γ_2 be O. We let the length of the altitude from O to BC be x, then the side length of γ_1 is $2\sqrt{3}x$ so the length of the altitude from O to HI is $(1+2\sqrt{3})x$.

   From this, we find that the ratio squared is:

$$\left(\frac{(1+2\sqrt{3})x}{x}\right)^2 = (1+2\sqrt{3})^2 = 1+4\sqrt{3}+12 = \boxed{13+4\sqrt{3}}.$$

   Proposed by Allen Wang

2. **Problem:** A decimal that terminates, like $1/2 = 0.5$ has a repeating block of 0. A number like $1/3 = 0.\overline{3}$ has a repeating block of length 1 since the fraction bar is only over 1 digit. Similarly, the numbers $0.0\overline{3}$ and $0.6\overline{5}$ have repeating blocks of length 1. Find the number of positive integers n less than 100 such that $1/n$ has a repeating block of length 1.

   Solution: Let $1/n$ have a repeating block r of length d with n being relatively prime to 10. By definition,

$$\frac{1}{n} = 0.\overline{r}$$

$$\frac{1}{n} = 0.r + 10^{-d} \cdot 0.\overline{r}.$$

$$\frac{10^d}{n} = r + 0.\overline{r}.$$

$$\frac{10^d}{n} = r + \frac{1}{n}.$$

$$10^d = nr + 1$$

$$r = \frac{10^d - 1}{n}$$

   In particular, the last equation tells us that n must divide $10^d - 1$ evenly. Since the repeating block has a length d of 1, we see that n must divide 9. So n equals 3 or 9. But recall we set that the condition that n is relatively prime to 10. In fact, all multiples of 2 and 5 of n also work. The proof is simple and left to the reader as an exercise.

It remains to count the multiples of 3 and 9 with 2 and 5 below 100.

For 3, these are: $3, 3 \cdot 5, 3 \cdot 5^2, 3 \cdot 2, 3 \cdot 2 \cdot 5, 3 \cdot 2^2, 3 \cdot 2^2 \cdot 5, 3 \cdot 2^3, 3 \cdot 2^4, 3 \cdot 2^5$.

For 9, these are: $9, 9 \cdot 5, 9 \cdot 2, 9 \cdot 2 \cdot 5, 9 \cdot 2^2, 9 \cdot 2^3$.

There are $\boxed{16}$ values of n.

Proposed by Allen Wang

3. **Problem:** For how many positive integers n between 1 and 2017 is the fraction

$$\frac{n+6}{2n+6}$$

irreducible? (Irreducibility implies that the greatest common factor of the numerator and the denominator is 1.)

Solution: If the fraction were reducible, then a number g would divide both the numerator and denominator. So the numerator N and the denominator D can be written as $N = gN'$ and $D = gD'$. But $N - D = gN' - gD' = g(N' - D')$, which is divisible by g.

So we count the number of reducible fractions. If the fraction were reducible, then g must divide their difference, $2n + 6 - (n + 6) = n$. But since g divides n and g divides $n + 6$, g divides that difference also, $n + 6 - n = 6$. So g is a factor of 6. A quick check shows that the fraction reduces whenever n is not divisible by $2, 3$, or 6. So n leaves a remainder of 1 or 5 when it is divided by 6.

In the interval $6k + 1, 6k + 2, 6k + 3, 6k + 4, 6k + 5, 6k + 6$, only two values for n work. From 1 to 2016, we have $\frac{2016}{6} = 336$ intervals, two numbers work in each giving $2 \cdot 336 = 672$ values of n. We add 1 for the lonely 2017 to get the final answer of $\boxed{673}$.

Proposed by Allen Wang

Round 6

1. **Problem:** Consider the binary representations of $2017, 2017 \cdot 2, 2017 \cdot 2^2, 2017 \cdot 2^3, \cdots, 2017 \cdot 2^{100}$. If we take a random digit from any of these binary representations, what is the probability that this digit is a 1?

Solution: We begin by converting 2017 to binary: since $2017 = 1024 + 512 + 256 + 128 + 64 + 32 + 1 = 11111100001_2$. Every time we multiply this number by 2, we add another 0 to the end of this number. So $2017 \cdot 2^{100}$ is the same number but with 100 zeroes appended.

2017 has 11 digits, while $2017 \cdot 2^{100}$ has 111 digits. Summing this arithmetic sequence gives a total of

$$\left(\frac{11 + 111}{2}\right) \cdot 101 \text{ digits.}$$

But the amount of 1s in each number is always 7. Since there are 101 numbers, there are $101 \cdot 7$ ones. Therefore, our desired probability is

$$\frac{7 \cdot 101}{\left(\frac{11+111}{2}\right) \cdot 101} = \boxed{\frac{7}{61}}.$$

Proposed by Akshay Gowrishankar and Richard Huang

2. **Problem:** Aaron is throwing balls at Carlson's face. These balls are infinitely small and hit Carlson's face at only 1 point. Carlson has a flat, circular face with a radius of 5 inches. Carlson's mouth is a circle of radius 1 inch and is concentric with his face. The probability of a ball hitting any point on Carlson's face is directly proportional to its distance from the center of Carlson's face (so when you are 2 times farther away from the center, the probability of hitting that point is 2 times as large). If Aaron throws one ball, and it is guaranteed to hit Carlson's face, what is the probability that it lands in Carlson's mouth?

   Solution: Let every point on Carlson's face correspond to a point (x, y) in the plane. We can toss this plane into 3-dimensional space, and let (x, y, z) correspond to the point (x, y) and its probability of being hit, z.

   We see that the locus of points x, y, z form the complement of a cone. The graph of the equation is $z = \sqrt{x^2 + y^2}$.

   Moreover, the probability of choosing a point (x, y) on Carlson's face is the probability of choosing a point in the cone complement that has a x and y component of (x, y). The probability is thus equal to the ratio of two 3-dimensional objects with a side-length ratio of $1 : 5$. So the desired probability is $\boxed{\frac{1}{125}}$.

   Proposed by Akshay Gowrishankar

3. **Problem:** he birth years of Atharva, his father, and his paternal grandfather form a geometric sequence. The birth years of Atharva's sister, their mother, and their grandfather (the same grandfather) form an arithmetic sequence. If Atharva's sister is 5 years younger than Atharva and all 5 people were born less than 200 years ago (from 2017), what is Atharva's mother's birth year?

   Solution: Let the birth year of Atharva's grandfather be x and the birth year of Atharva's father be an integer xr, where r is a rational number greater than 1. We can then express Atharva's birth year as xr^2.

   Note that if $r = \frac{p}{q}$, x is divisible by q^2, and xr^2 is divisible by p^2. We need to make the value of r close to 1; specifically, $r > \sqrt{\frac{2017}{1817}}$, since otherwise, xr^2 would be too large.

   In order to make r close to 1, we have to select a number r such that $q = p - 1$, since we can quickly see that there are no plausible values for p and q if $p - q > 1$.

   Therefore, we need to find consecutive integers p and q such that kp^2 and kq^2 are both in the interval [1817, 2017] for some positive integer k.

   If $k = 1$, then we have $p = 44$ and $q = 43$. This turns out to be the only solution since if k, $p = 31$ is the only value that yields a kp^2 in the interval and there would not be a possible value for q. $k = 3$, $k = 4$, and $k = 5$ also yield only 1 value for p and no possible values for q and if $k > 3$, then r would be too small.

Therefore, $r = \frac{44}{43}$, Atharva's birth year is $44^2 = 1936$, and Atharva's grandfather's birth year is $43^2 = 1849$.

Atharva's sister is 5 years younger than him; therefore she is born in 1941. If her grandfather, her mother, and her form an arithmetic sequence, then her mother's birth year is the average of her grandfather's and her's. Therefore, her mother's birth year is $\frac{1849+1941}{2} = \boxed{1895}$.

Proposed by Akshay Gowrishankar

Round 7

1. **Problem:** A function f is called an "involution" if $f(f(x)) = x$ for all x in the domain of f and the inverse of f exists. Find the total number of involutions f with domain of integers between 1 and 8 inclusive.

   Solution: If $f(x) = x$ then x is called a fixed point. In an involution, all elements that are not paired together by the equation $f(x) = y, f(y) = x$ must be fixed points. We do case work on the number of fixed points:

   - 8 fixed points: there is one 1 such function.
   - 6 fixed points: we choose the 1 pair in $\binom{8}{2}$ ways.
   - 4 fixed points: we choose 2 pairs and divide for the ordering of the pairs in $\frac{1}{2!}\binom{8}{2}\binom{6}{2}$ ways.
   - 2 fixed points: we choose 3 pairs and divide for ordering of the pairs in $\frac{1}{3!}\binom{8}{2}\binom{6}{2}\binom{4}{2}$ ways.
   - Finally, we choose 4 pairs and divide for the ordering of the pairs in $\frac{1}{4!}\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2}$ ways.

   Adding the binomials from the expressions above, we find that the total number of involutions is $\boxed{764}$.

   Proposed by Allen Wang

2. **Problem:** The function $f(x) = x^3$ is an odd function since each point on $f(x)$ corresponds (through a reflection through the origin) to a point on $f(x)$. For example the point $(-2, -8)$ corresponds to $(2, 8)$.

   The function $g(x) = x^3 - 3x^2 + 6x - 10$ is a "semi-odd" function, since there is a point (a, b) on the function such that each point on $g(x)$ corresponds to a point on $g(x)$ via a reflection over (a, b). Find (a, b).

   Solution: Note that

   $$f(0) = -10$$

   and

   $$f(1) = 1 - 3 + 6 - 10 = -6.$$

   So $(0, -10)$ and $(1, -6)$ are points on $f(x)$.

   The reflections of these two points across (a, b) are $(2a, 2b+10)$ and $(2a-1, 2b+6)$, respectively.

   The reflection points must be on the function since $f(x)$ is semi-odd.

   So

   $$(2a)^3 - 3(2a)^2 + 6(2a) - 10 = 2b + 10$$
   $$(2a-1)^3 - 3(2a-1)^2 + 6(2a-1) - 10 = 2b + 6$$

We chug through the system and get two solutions: $(a, b) = (1, -6)$ and $(a, b) = (\frac{1}{2}, -8)$.

But the latter solution is not on the function, so $\boxed{(1, -6)}$ is the answer.

Proposed by Allen Wang

3. **Problem:** A permutations of the numbers $1, 2, 3, 4, 5$ is an arrangement of the numbers. For example, 12345 is one arrangement, and 32541 is another arrangement. Another way to look at permutations is to see each permutation as a function from $\{1, 2, 3, 4, 5\}$ to $\{1, 2, 3, 4, 5\}$. For example, the permutation 23154 corresponds to the function f with $f(1) = 2$, $f(2) = 3$, $f(3) = 1$, $f(5) = 4$, and $f(4) = 5$, where $f(x)$ is the xth number of the permutation.

   But the permutation 23154 has a cycle of length three since $f(1) = 2$, $f(2) = 3$, $f(3) = 1$, and cycles after 3 applications of f when regarding a set of 3 distinct numbers in the domain and range. Similarly the permutation 32541 has a cycle of length three since $f(5) = 1$, $f(1) = 3$, and $f(3) = 5$.

   In a permutation of the natural numbers between 1 and 2017 inclusive, find the expected number of cycles of length 3.

   Solution: Consider the cycle $1 \to 2 \to 3$. Let X_1 be the variable that equals 1 if this cycle exists in a permutation and 0 if this cycle does not exist in a permutation. We see that the expected number of $1 \to 2 \to 3$ cycles in a random permutation is equal the value of X_1 averaged over all permutations. This average, by definition, is the probability that a $1 \to 2 \to 3$ cycle exists in a permutation.

   In a $1 \to 2 \to 3$, $f(1) = 2, f(2) = 3$, and $f(3) = 1$. All three function values are determined. So the probability of this happening is $\frac{1}{2017 \cdot 2016 \cdot 2015}$.

   Note that this is the expected value of X_1, similarly we can define X_1' be the indicator variable for the cycle $1 \to 3 \to 2$, let X_2 be the indicator variable for the cycle $1 \to 2 \to 4$, and so on. The expected value of cycles of length 3 is the sum of the expected value of all of these indicator variables.

   By symmetry, the expected value of each of these indicator variables is equal to

$$X_1 = \frac{1}{2017 \cdot 2016 \cdot 2015}.$$

   The number of indicator variables equals the total number of cycles of length 3. A cycle of length 3 is specified by 2 "parameters," a combination of 3 numbers and a "direction." For example the combination $\{1, 2, 3\}$ can have 2 directions, producing 2 cycles (these two directions are indicated by X_1 and X_1'). Therefore the total number of cycles is $2\binom{2017}{3}$.

   In particular,

$$\mathbb{E}[\text{Number of Cycles of Length 3}] = \sum_X = 2\binom{2017}{3} \cdot \frac{1}{2017 \cdot 2016 \cdot 2015} = \boxed{\frac{1}{3}}.$$

   Note that our answer did not depend on the number 2017 at all.

   Proposed by Allen Wang

Round 8

Problem: Find the number of characters in the problems on the accuracy round test. This does not include spaces and problem numbers (or the periods after problem numbers). For example, "1. What's 5 + 10?" would contain 11 characters, namely "W," "h," "a," "t," "'," "s," "5," "+," "1," "0," "?". If the correct answer is c and your answer is x, then your score will be

$$\max\left\{0,\ 13-\left\lceil\frac{|x-c|}{100}\right\rceil\right\}.$$

Solution: The correct answer is $\boxed{1586}$ characters in the accuracy round. One can count the number of letters in this problem and multiply it by the amount of questions for a decent estimation.

Proposed by Akshay Gowrishankar